Delay Analysis of Three-State Markov Channels

[1]Jian Zhang, [1]Zhiqiang Zhou, [2]Tony T. Lee, and [1]Tong Ye

[1]State Key Lab of Advanced Optical Communications and Networks
Shanghai Jiao Tong University, Shanghai, China
[2]Chinese University of Hong Kong (Shenzhen), Shenzhen, China

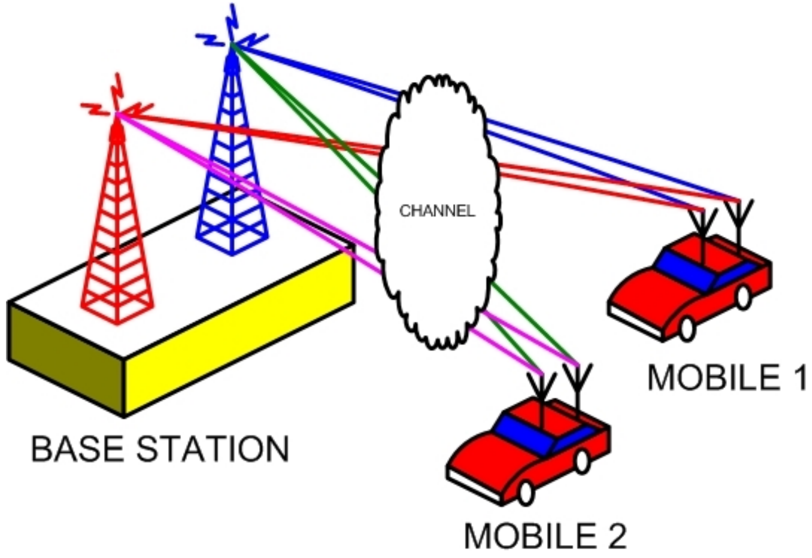
Aug. 22, 2017 @ QTNA 2017

Outline

- Background and Problem
- Hybrid embedded Markov Chain
- Mean waiting time
- Conclusion

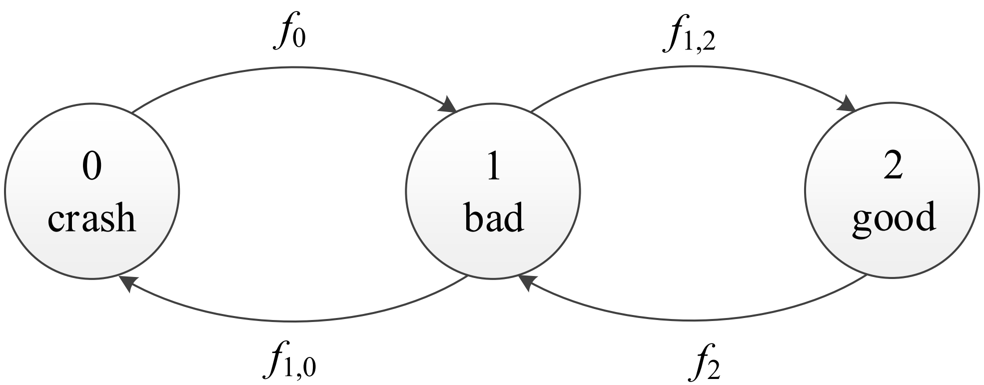
Background

- Wireless channel suffers from slow/fast fading and transmission rate changes during packet service
- Our goal is to figure out what factors affect delay performance



Three-state Markov Channel Model

- Rate fluctuation of wireless channel can be capture by a three-stage Markov Chain
  - μ_j: service rate in state j ($\mu_0 = 0, \mu_1 < \mu_2$)
  - π_j: Steady state probability that the channel is in state j



$$\boldsymbol{\pi Q} = \mathbf{0}$$

$$\sum_i \pi_i = 1 \quad \Longrightarrow$$

$$\boldsymbol{Q} = \begin{pmatrix} -f_0 & f_0 & 0 \\ f_{1,0} & -f_1 & f_{1,2} \\ 0 & f_2 & -f_2 \end{pmatrix} \qquad \boldsymbol{D} = \begin{pmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_2 \end{pmatrix}$$

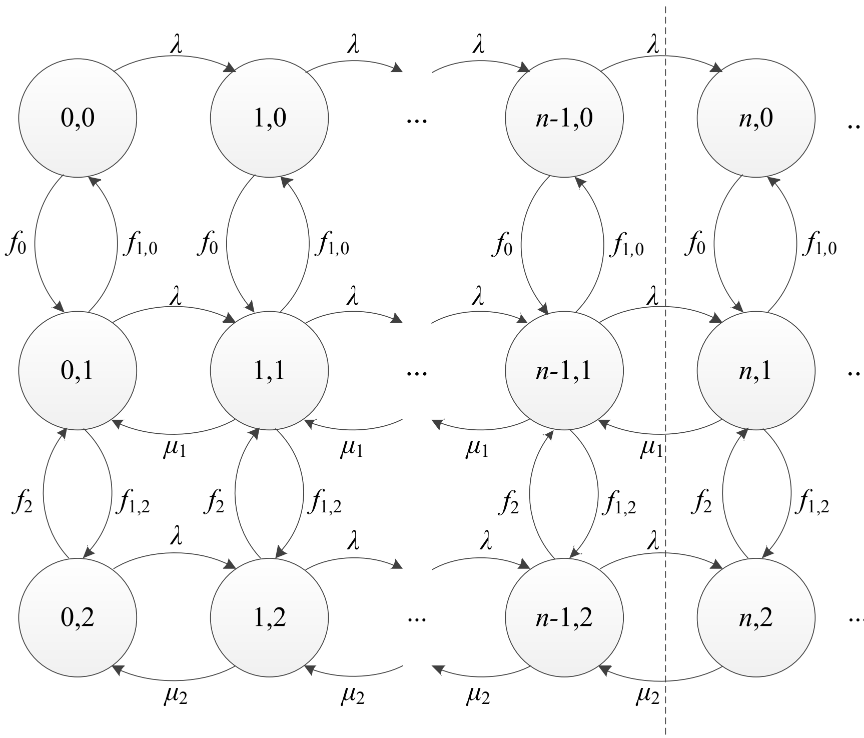
$$\pi_0 = \frac{f_{1,0} f_2}{f_0 f_2 + f_2 f_{1,0} + f_0 f_{1,2}}$$

$$\pi_1 = \frac{f_0 f_2}{f_0 f_2 + f_2 f_{1,0} + f_0 f_{1,2}}$$

$$\pi_2 = \frac{f_0 f_2}{f_0 f_2 + f_2 f_{1,0} + f_0 f_{1,2}}$$

M/MMSP/1 queuing model

- If traffic input is a Poisson traffic with rate λ, wireless communication system can be by an M/MMSP/1 model
  - $(X(t), Y(t))$: (number of packets, channel state) at time t



$\Longrightarrow$ Steady state probability
$$p_{n,j} = \lim_{t\to\infty} P\{X(t) = n, Y(t) = j\}$$

$\Longrightarrow$ Mean Delay: An numerical Solution

Outline

- Background and Problem
- Hybrid embedded Markov Chain
- Mean waiting time
- Conclusion

Feature of M/MMSP/1 Queue

- Service time of packets are not independently and identically distributed. It depends on the state in which the packet start being served [1].
  - Start service in larger service rate state will lead to a smaller service time.
  - The start service state of one packet is dependent on the start service state of last packet.

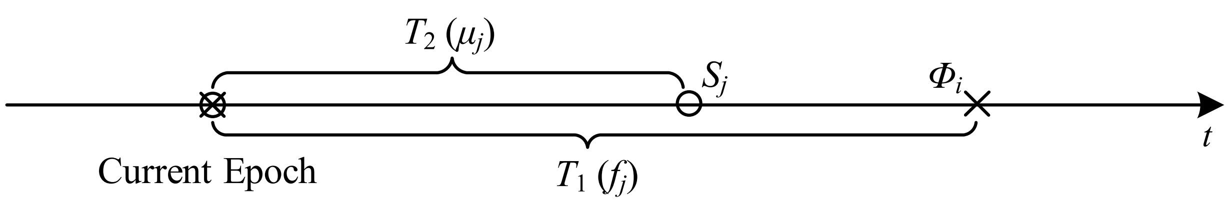
- We use hybrid embedded Markov chain to describe the channel state transition during the service of packets.

[1] Mahabhashyam, S R., Natarajan G.: On queues with Markov modulated service rates. Queueing Systems 51(1), 89-113(2005).

Hybrid Embedded Points

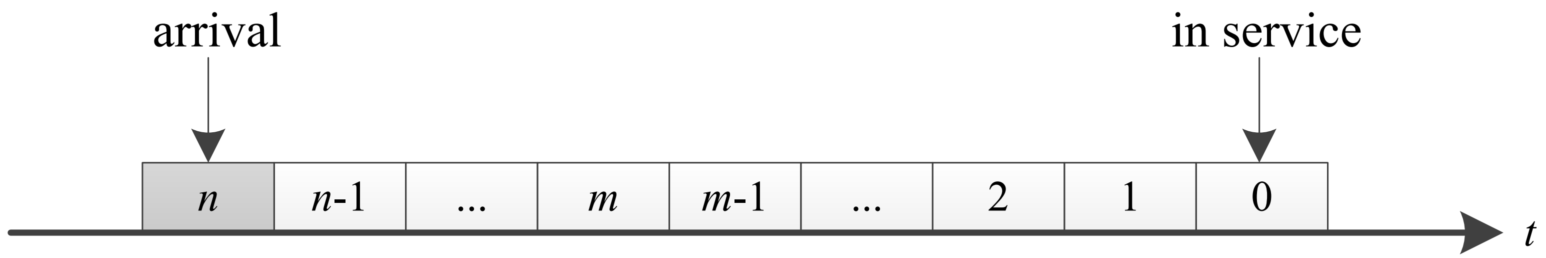
- Φ_j: Epoch when state transits, after which channel state is j
- S_j : Epoch when service starts, after which channel state is j

- If current epoch is an embedded point with state j
  - Probability that next embedded point is $\Phi_i (i \neq j)$: $f_{j,i}/(\mu_j + f_j)$;
  - Probability that next embedded point is S_j: $\mu_j/(\mu_j + f_j)$;
  - Holding time, time from current embedded point to next embedded point, follows an exponential distribution with parameter $\mu_j + f_j$.



Probability at Embedded Points

- $\hat{\pi}_{n,j}(m) = \Pr\{\text{start}-\text{service state of } m^{th} \text{ packet is } j | \text{an arrival sees } n \text{ packets in the system}\}$
- $\hat{\varphi}_{n,j}(m) = \Pr\{\text{during service of } m^{th} \text{ packet, channel transits into state } j | \text{an arrival sees } n \text{ packets in the system}\}$



Start Service Probability

- State equations of conditional start service probability
  - $\hat{\pi}_{n,j}(m) = \frac{\mu_j}{\mu_j+f_j}\left(\hat{\pi}_{n,j}(m-1) + \hat{\varphi}_{n,j}(m-1)\right)$
  - $\hat{\varphi}_{n,j}(m) = \sum_{i \neq j} \frac{f_{i,j}}{\mu_i+f_i}\left(\hat{\pi}_{n,i}(m) + \hat{\varphi}_{n,i}(m)\right)$

- Matrix form of conditional start service probability

$$\hat{\pi}_n(m) = \left((D-Q)^{-1}D\right)^T \hat{\pi}_n(m-1)$$

where $\hat{\pi}_n(m) = \left(\hat{\pi}_{n,0}(m), \hat{\pi}_{n,1}(m), \hat{\pi}_{n,2}(m)\right)^T$

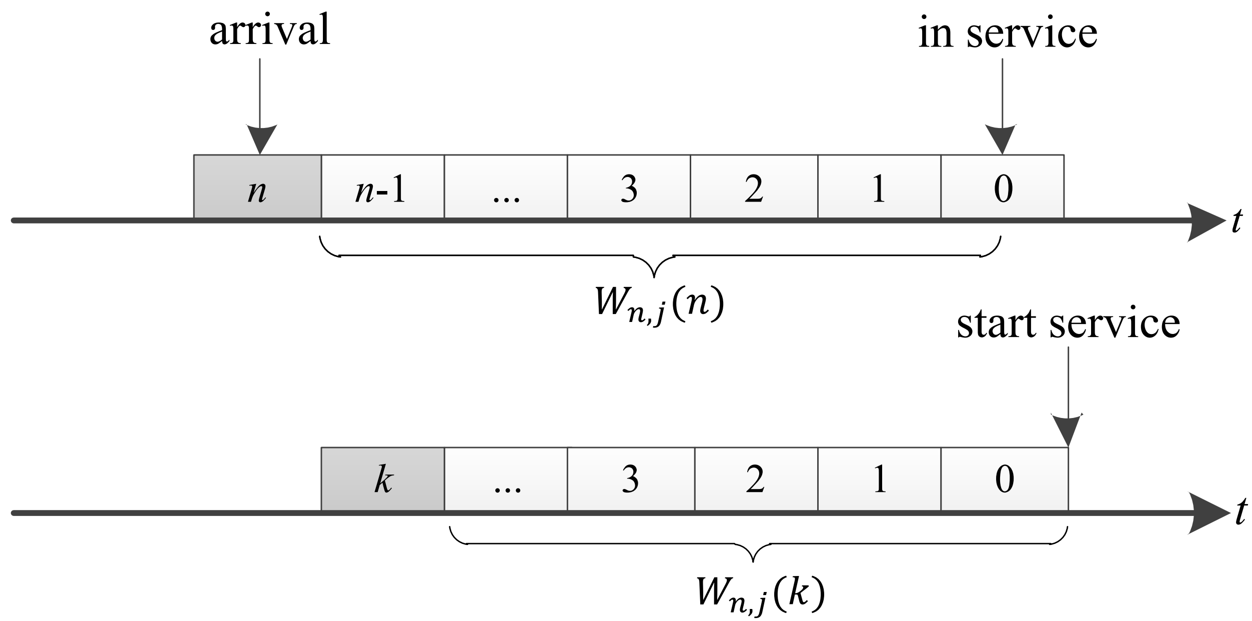
- Start service probability
  - $\hat{\pi}_j = \sum_{n=0}^{\infty} p_n \, \hat{\pi}_{n,j}(n)$

Outline

- Background and Problem
- Hybrid embedded Markov Chain
- Mean waiting time
- Conclusion

Residual waiting time at embedded points

- A new arrival packet sees n packets in the buffer:
  - $W_{n,j}(k)$: The residual waiting time from the epoch when this new arrival becomes the k^{th} packet in the buffer while the channel is in state j to the epoch when it becomes an HOL packet.
  - $V_{n,j}(k)$: The residual waiting time from the epoch when the channel transits to state j while this new packet is the k^{th} packet in the buffer to the epoch when it becomes an HOL packet.



Mean Waiting Time

- State equations of conditional waiting time
  - $W_{n,j}(k+1) = \frac{\mu_j}{\mu_j+f_j}\left(\frac{1}{\mu_j+f_j} + W_{n,j}(k)\right) + \sum_{i\neq j}\frac{f_{j,i}}{\mu_j+f_j}\left(\frac{1}{\mu_j+f_j} + V_{n,i}(k+1)\right)$
  - $V_{n,j}(k+1) = \frac{\mu_j}{\mu_j+f_j}\left(\frac{1}{\mu_j+f_j} + W_{n,j}(k)\right) + \sum_{i\neq j}\frac{f_{j,i}}{\mu_j+f_j}\left(\frac{1}{\mu_j+f_j} + V_{n,i}(k+1)\right)$

- Matrix form of conditional waiting time

$$\boldsymbol{W}_n(k+1) = (\boldsymbol{D}-\boldsymbol{Q})^{-1}\boldsymbol{D}\boldsymbol{W}_n(k) + (\boldsymbol{D}-\boldsymbol{Q})^{-1}\mathbf{1}$$

where

$$\boldsymbol{W}_n(k) = \left(W_{n,0}(k), W_{n,1}(k), W_{n,2}(k)\right)^T$$

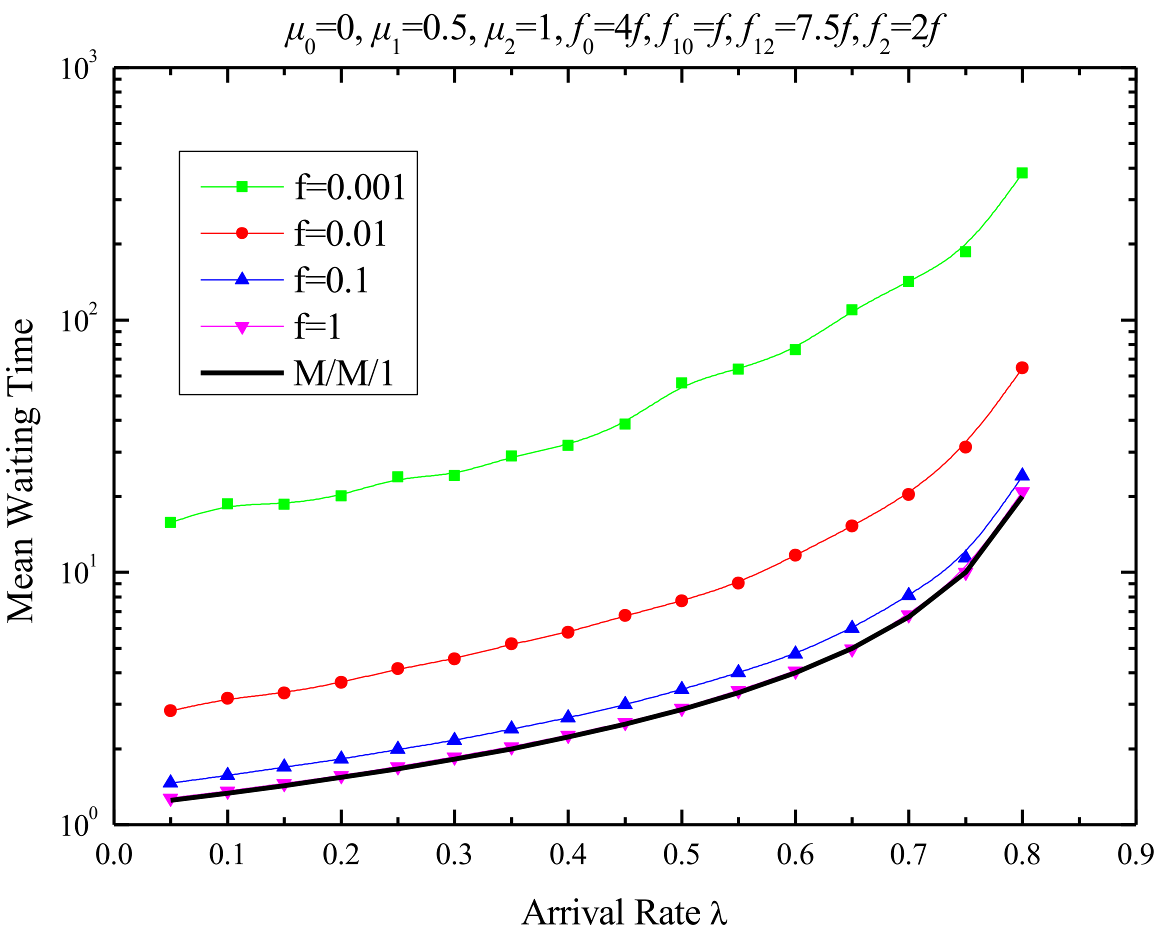
Mean Waiting Time

- Mean waiting time
  - $W = \sum_{j=0}^{2} \sum_{n=1}^{\infty} W_{n,j}(n) p_{n,j}$

$$= \frac{\frac{1}{\hat{\mu}}\lambda E[T] + \frac{1}{1-\beta}\sum_{j=0}^{2} E[T_j](\pi_j - \hat{\pi}_j) - \frac{\beta}{1-\beta}\frac{1}{f_0}(\pi_0 - \hat{\pi}_0)}{1-\frac{\lambda}{\hat{\mu}}}$$

- $\beta = \mu_1\mu_2/(\mu_1\mu_2 + \mu_1 f_2 + \mu_2 f_{1,2})$ is one of the 3 eigenvalues of $(\boldsymbol{D} - \boldsymbol{Q})^{-1}\boldsymbol{D}$.
  - $\beta \to 0$: state transition rate is much larger than service rate.
  - $\beta \to 1$: state transition rate is much smaller than service rate.

Simulation result



Conclusions

- Many problems in communication and computer networks can be modeled as M/MMSP/1 queueing model with several states, of which the numerical solution provides little physical insight
- With the help of hybrid embedded Markov chain, we obtain a structural solution and find that delay is influenced by state transition rate significantly
- Our approach can be easily extended to other finite-state M/MMSP/1 queueing model

Thank You!