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Optimum Transmission Window for EPONs with Limited Service

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ABSTRACT This paper studies the Ethernet Passive Optical Network (EPON) with limited service. The transmission window (TW) is limited in this system to guarantee a bounded delay experienced by disciplined users and to constrain malicious users from monopolizing the transmission channel. Thus, selecting an appropriate TW size is critical to the performance of EPON with limited service discipline. In this paper, we develop an M/G/1 queueing model with vacation times and limited-service discipline to investigate the impact of TW size on packet delay. A distinguishing feature of this model is that there are two queues in the buffer of each optical network unit (ONU): one queue is inside the gate and the other one is outside the gate, from which we derive the generalized formula of mean waiting time. Furthermore, based on the service level agreements of users and the Chernoff bound of queue length, we provide a simple rule to determine an optimum TW size for limited-service EPONs. Analytic results reported in this paper are all verified by simulations.

INDEX TERMS Ethernet Passive Optical Network (EPON), Limited Service, M/G/1.

I. INTRODUCTION

THE ever-growing Internet traffic generated by emerging services, including video on demand, remote e-learning, and online gaming, continues to exacerbate the last mile bottleneck problem [1]. Ethernet Passive Optical Network (EPON) has been considered as an attractive solution to this problem due to its low cost, large capacity and ease of upgrade to higher bit rates [2]. It has been widely deployed in many access networks such as Fiber-To-The-Home (FTTH), Fiber-To-The-Building (FTTB) and Fiber-To-The-Curb (FTTC) [3]–[5]. In particular, today FTTH is a predominate deployment because it provides high bandwidth for each end-user [6], [7].

A typical EPON is plotted in FIGURE 1. An EPON is a point-to-multipoint network, where one optical line terminal (OLT) in the central office is connected to multiple optical network units (ONUs) located at the users' premises via an optical passive splitter. In the downstream direction, the OLT broadcasts the packets to all ONUs, and each ONU only accepts the packets destined to it. In the upstream direction, the OLT schedules the ONUs to share the bandwidth in a time division multiplexing (TDM) manner. The OLT assigns transmission windows (TWs) to each ONU by sending

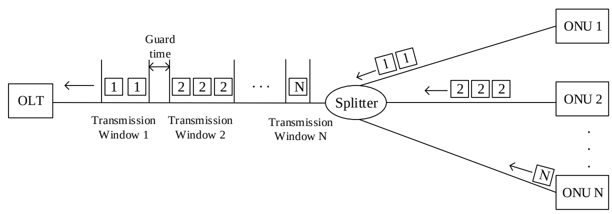

FIGURE 1: Upstream transmission in the EPON.

GATE messages in a round-robin fashion. Upon receiving the GATE message, the ONU transmits upstream data in the allocated TW. The number of packets that the ONU can send during a TW is called the $TW size$ in this paper. After data transmission, the ONU generates a REPORT message to inform the OLT of its buffer status [2]. The TWs of two successive ONUs are separated by a guard time to avoid data overlapping. The sizes of TWs that the OLT allocates to each ONU depend on the service discipline that the OLT adopts.

The gated-service discipline has been widely studied in previous works [8]–[15]. In gated service, each ONU is authorized to transmit the amount of data that it requests in the REPORT [16]. Thus, the gated service may lead to the phenomenon called the "capture effect" [17], when an

ONU with heavy traffic monopolizes the upstream channel for a long time and transmits an excessive amount of data. This is especially true in the application scenario of FTTH, where the ONU is installed in the user's home and the user can easily manipulate the input traffic rate of the ONU. The capture effect imposes a large delay on other ONUs and, thus, impairs the quality of service (QoS) of other ONUs.

The limited-service discipline was then used to solve this problem [8]. With limited service, EPON users sign a service level agreement (SLA) with the network operator to specify the upstream traffic rate, and the OLT typically sets a limit of the maximum TW size to guarantee the QoS of each ONU according to the signed SLA. In this case, the TW size allocated by the OLT to an ONU is not larger than this preset maximum TW size, and thus the "capture effect" can be suppressed, even if some users inject more traffic than what they have agreed to with the network operator.

The selection of the maximum TW size is a critical choice in the limited-service EPON. On one hand, if the maximum TW size is set too small, the backlog of the ONUs cannot be cleaned up quickly and the upstream bandwidth is wasted by many guard times and REPORT messages. In this case, the ONU will suffer from a large delay. On the other hand, if the maximum TW size is set too large, the capture effect cannot be suppressed effectively. An extreme case is that the limited-service discipline will change to the gated-service discipline when the limit goes to infinity.

In the literature, only a few previous works have studied the selection of the maximum TW size via simulations. The impact of the maximum TW size on delay performance of an ONU is discussed in [18], in which the author points out that the maximum TW size for each ONU can be fixed based on the SLA, but doesn't provide any concrete scheme for the selection of the maximum TW size. The aim of our paper is to develop a systematic method to select a proper maximum TW size for limited-service EPONs.

The upstream transmission process of each ONU can be described by a vacation queueing system, in which each TW of the ONU is considered as a busy period while the time between two successive TWs of the ONU is treated as a vacation period. In general, the modeling of a vacation queueing system with limited-service discipline is quite difficult. As we show in Section II, the traditional method to solve the mean waiting time is via a complex embedded Markov chain, in which the embedded points are defined at the epochs when a packet departs and when a vacation finishes [19], [20]. Thus, there is still no simple way to derive the mean waiting time for limited-service EPONs, and only an approximate result is currently available in [8].

In this paper, we focus on the investigation of limited-service EPONs for FTTH applications. Our goal is to develop an insightful model to describe the delay performance of limited-service EPONs and find a systematic method of selecting the maximum TW size for each ONU based on the SLA.

First, we propose a simple method to obtain a generalized formula of the mean waiting time. The distinguishing feature of our modeling approach is that we divide the buffer of the limited-service queue into two virtual queues: one is inside the gate and the other one is outside the gate. Based on this approach, we find a key parameter, the average number of whole vacations that a packet has to experience before it receives service, so that we can obtain a generalized formula of the mean waiting time simply by extending the geometric model in [21]. This formula clearly indicates how the mean waiting time depends on the parameters, such as service time, vacation time, and the first and second moments of the number of packets served in a busy period.

Next, we apply the Chernoff bound of queue length to select the optimum TW size. According to the SLA, the delay performance of an ONU shouldn't be influenced by the TW size limit if its traffic rate does not exceed the subscribed rate. Thus, the criterion of selecting the optimum TW size is to choose the smallest integer that makes the probability of the queue length exceeding the TW size limit negligible. That is, when an ONU operates in the subscripted region, its buffer can be emptied with a high probability at the end of every busy period. Otherwise, the ONU will suffer from a large delay when the input traffic rate exceeds the subscribed rate. Our specific contributions are summarized as follows:

1. We develop a simple method to derive a generalized formula of mean waiting time for an M/G/1 queue with vacation time and limited-service discipline, which includes the mean waiting times of two virtual queues: one is inside the gate and the other one is outside the gate.
2. We provide a simple rule to determine a proper optimum TW size for ONUs of the limited-service EPON based on their SLAs, which is proved to be effective by simulations.

The remainder of this paper is organized as follows. In Section II, we describe the works related to the study of limited-service EPONs. In Section III, we provide an overview of the polling process between the OLT and ONUs of a limited-service EPON and derive the mean waiting time. Section IV discusses the method of selecting the optimum TW size and demonstrates the performance of a limited-service EPON under the selected TW size. Section V concludes this paper.

II. RELATED WORKS

A limited-service EPON is a polling system where a single server manages a set of queues in a cyclic order, using limited-service discipline. On the other hand, each ONU is actually a vacation queueing system with limited-service discipline, where each duration that the OLT serves the queue of the ONU can be treated as a busy period while the duration that OLT polls other ONUs can be considered as a vacation. A large number of previous works studied the different types of limited-service vacation queueing systems, including the exhaustive type [19], [22]–[24] and the gated type [20], [25].

The exhaustive-type k-limited vacation queueing systems were studied in [19], [22]–[24], where the server takes a vacation when either a queue has been emptied or a predefined number of k customers have been served during the visit. In [19], the distributions of queue length, waiting time and busy period were obtained by using the embedded Markov chain, in which the embedded points are defined at the epochs when a packet departs or a vacation finishes, and a combination of the supplementary variables and sample biasing techniques. This kind of method is quite complex. In [22], [23], the authors used matrix-analytic techniques to iteratively calculate the queue length distribution. In [24], a polling system with two priority queues and k-limited service discipline was analyzed, where the high priority queue is served with queue length dependent service time while the low priority queue is served with constant service time. The high priority queue length distribution at departure instants was derived by the embedded Markov chain. However, these models cannot be directly applied to limited-service EPONs, in which the OLT only serves the packets that arrived before the last REPORT message of an ONU up to a predefined number, regardless if the buffer is empty or not.

The gated-type k-limited service vacation queueing systems were considered in [20], [25], where the server manages at most k customers that present at a queue upon visiting and then begins a vacation. A queueing model based on an embedded Markov chain was developed in [20], [25] to derive the Laplace-Stieltjes transforms of waiting time and busy period distributions, but the computation is too complex to give a clear physical insight into the performance of the entire system. To resolve this problem, a simple geometric approach was proposed in [21] to obtain the mean waiting time, but this approach can only solve a special case when the user is allowed to transmit one packet in each busy period.

Each ONU of a limited-service EPON is a kind of gated-type limited-service vacation queueing system [20]. However, only a few works in literature were devoted to the modeling of limited-service EPONs. In [8], the authors gave an approximate expression of mean waiting time for a limited-service EPON under the assumption that the maximum TW size in terms of time (instead of the number of packets) is quite large, which is actually similar to the analysis of the gated-service EPON. In [10], an approximate mean delay of limited-service EPONs is derived by using a discrete Markov chain, which is invalid when the traffic load is high.

In summary, no previous work has obtained a useful formula for mean waiting time in general limited-service EPONs where the maximum TW size is finite and larger than one. No previous work has discussed how to select a proper maximum TW size for each ONU of limited-service EPONs, which will be studied in this paper.

III. MODELING LIMITED-SERVICE EPONS

In this section, we analyze the mean waiting time of an ONU in limited-service EPONs. In Section III-A, we introduce the working process of the limited-service EPON and show that each ONU in the EPON can be modelled by a limited-service vacation queueing system. In Section III-B, we derive the mean waiting time of a general limited-service vacation queueing system, based on which we obtain the mean waiting time of the packet in the limited-service EPON in Section III-C.

A. WORKING PROCESS OF LIMITED-SERVICE EPON

In an EPON system with N ONUs that adopts the limited-service discipline, the packets waiting in the buffer of each ONU are divided into two groups by a fictitious gate. The number of packets inside the gate is bounded by the maximum TW size, denoted by M. An arrival packet first waits outside the gate and then enters the gate before it can be transmitted. As FIGURE 2 illustrates, the buffer status is represented by a two-tuple state (n, m), where n is the number of packets waiting outside the gate, and m is that waiting inside the gate. The number n increases by 1 upon a new arrival, and m decreases by 1 when a packet inside the gate begins to be transmitted by the ONU.

FIGURE 2 plots the polling process of an EPON, where $N = 2$ and $M = 3$. The OLT employs a 64-Byte GATE to notify an ONU about the start time and the length of each allocated TW. Upon receiving the GATE message, the ONU transmits all packets inside the gate during the TW. At the end of the TW, the ONU sends a 64-Byte REPORT to the OLT, which reports the number of packets waiting outside the gate. According to the number n stated in the REPORT, the OLT determines the TW size for this ONU in the next cycle, which equals the smaller of M and n. Thus, the message REPORT offers the admission for packets waiting outside the gate to enter the gate.

After ONU 1 issued the first REPORT, as FIGURE 2 shows, the buffer state changes from $(2, 0)$ to $(0, 2)$, which means two packets entered the gate. Then ONU 1 becomes idle while the OLT polls the next ONU. When the OLT finishes the polling of all other $(N - 1)$ ONUs, it sends a GATE message to ONU 1 again to repeat the process. To avoid data overlapping induced by the clock synchronization problem between the OLT and the ONUs [26], two successive TWs are separated by a guard time. As FIGURE 2 depicts, the constant interval G includes the guard time and the transmission time of a REPORT.

As FIGURE 2 shows, an ONU is busy with packet transmission during the TW, followed by a vacation period with a duration that is equal to the sum of NG and the TWs of other $(N - 1)$ ONUs if the propagation delay between the OLT and ONUs is not considered. At the end of each busy period, a predominate number of packets that an ONU reports to the OLT attributes to the number of arrivals during the vacation period before it. Hence, there are dependencies among TW sizes of ONUs. However, in the analysis of multiple access systems, such as Aloha or CSMA, this kind of dependency is weak and can be neglected when N is large [27], [28]. Thus, we can treat each ONU independently. We make the following assumptions in the modeling of limited-service EPONs:

A1. All ONUs in the EPON are statistically identical.
A2. The number of ONUs N is sufficiently large, say $N > 10$, such that the TWs of the ONUs can be considered

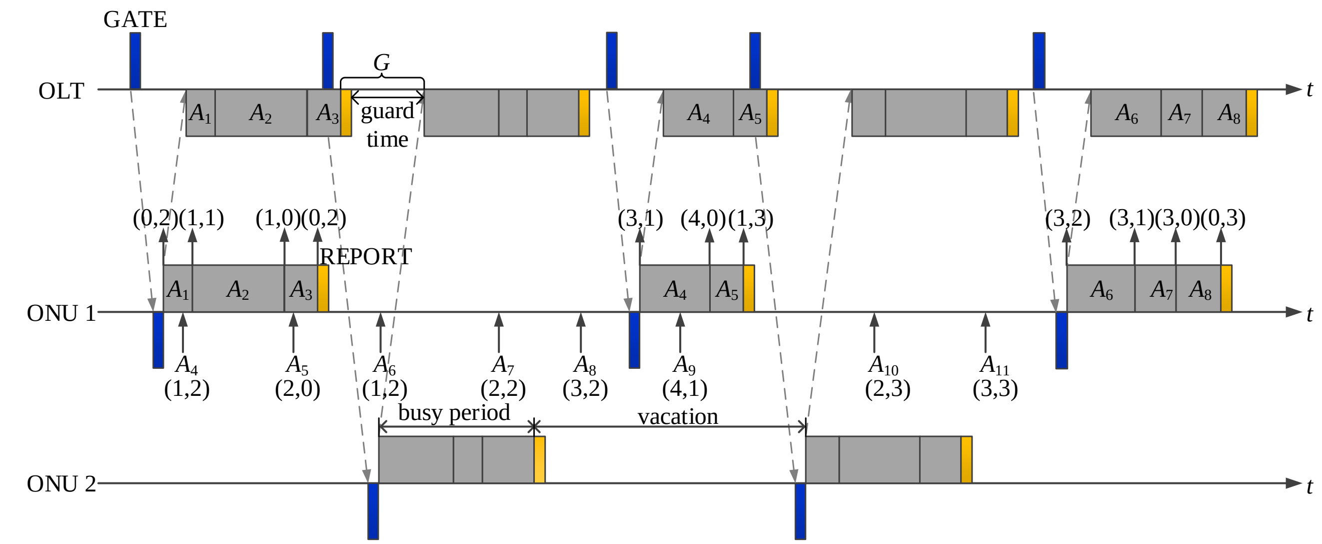

FIGURE 2: Polling process of an EPON where $N = 2$ and $M = 3$.

as i.i.d. random variables. That is justified, since the number of ONUs in a 10-Gb/s FTTH network is usually 32 or 64 in practice [29], [30], much larger than 10.

A3. The packet arrival process of the EPON is Poisson, as is the arrival process of each ONU.

A4. The packets are transmitted in a first-in-first-out (FIFO) manner, and the transmission times of the packets are i.i.d. random variables with a general distribution.

A5. The round-trip time (RTT) from the OLT to different ONUs is identical, as we consider the FTTH access network in this paper. To facilitate our analysis, we first ignore the RTT before Section IV-C, and call it back in Section IV-D.

Under these assumptions, each ONU can be considered as an M/G/1 queue with vacations and limited service. In Section III-B, we derive the mean waiting time of this queueing system, which provides the key to analyze the limited-service EPON in Section III-C.

B. M/G/1 QUEUE WITH VACATIONS AND LIMITED SERVICE

To facilitate our discussion, we adopt the following notations in the analysis of the M/G/1 queue with vacations and limited service:

(1) the packet-arrival rate, which is defined as the number of packets that arrive at the user per second, is λ packets/s,

(2) the service times of the packets X_1, $X_2, \cdots$ are i.i.d. random variables with the first moment $\overline{X}$ and the second moment $\overline{X^2}$, and

(3) the vacation times $V_1, V_2, \cdots$ are i.i.d. random variables with the first moment $\overline{V}$ and the second moment $\overline{V^2}$.

Under the limited-service discipline, up to M packets waiting outside the gate will enter the gate at the end of each busy period, and they will be served in the next busy period. After each busy period, the server takes a vacation. When the vacation terminates, the server returns to serve the packets if the buffer inside the gate is not empty; otherwise, the server takes another vacation.

A cycle starts at the end of a busy period. A cycle consists of a vacation period followed by another busy period. As FIGURE 3 illustrates, the i-th packet A_i may arrive at the system during a busy period or a vacation period. The following definitions pertaining to busy periods will be adopted in the derivation of mean waiting time of the packets:

B	A busy period.
K	The number of packets served in a busy period, where $K = 0, 1, 2, \cdots, M$.
B_k	A busy period, during which k packets are served, where $k = 0, 1, 2, \cdots, M$ (B_0 happens when a vacation finishes while the buffer inside the gate is empty).
b_k	The probability that a busy period is a B_k.
P_k	The probability that a packet is served in a B_k, where $k = 0, 1, 2, \cdots, M$.
Δ_i	The number of packets served ahead of the i-th packet A_i in the same busy period.

We need the following two lemmas to facilitate the derivation of the mean waiting time of packets.

Lemma 1. *The probability that the i-th packet A_i is served in a busy period B_k is given by*

$$P_k = \frac{kb_k}{\overline{K}} \tag{1}$$

Proof: Suppose there are θ_k busy periods B_k during a time interval $[0, T]$, where $k = 0, 1, 2, \cdots, M$. The probability b_k that a busy period is a B_k is defined by

$$b_k = \lim_{T\to\infty} \frac{\theta_k}{\sum_{k=0}^{M} \theta_k}$$

During the time interval $[0, T]$, the number of packets that are served in all θ_k busy periods B_k is $k\theta_k$, and the total

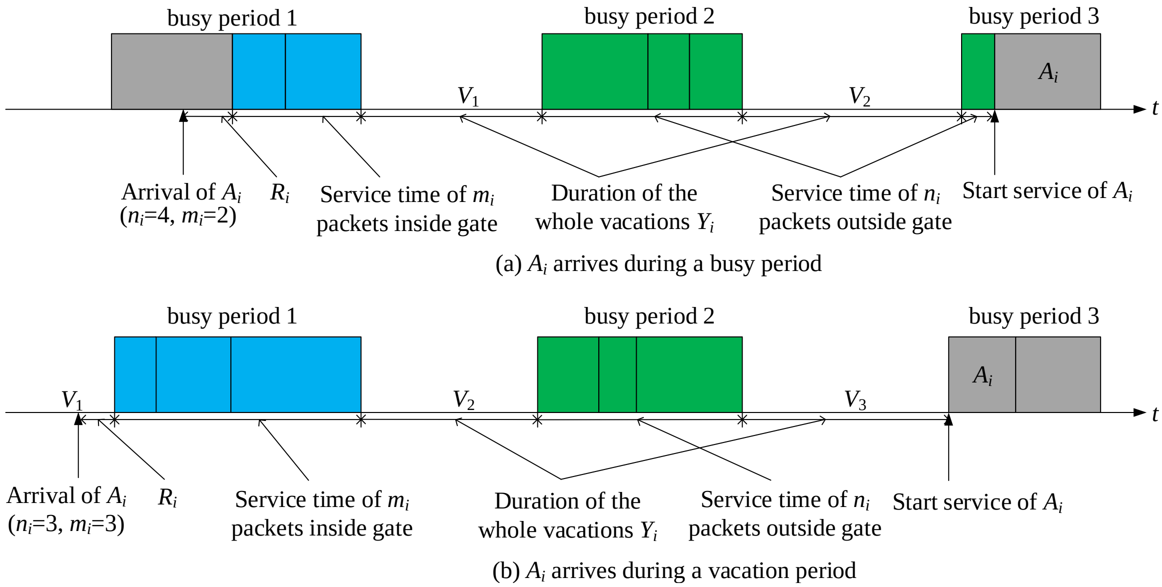

FIGURE 3: Waiting time of the i-th packet A_i, where $M = 3$.

number of packets served in the interval $[0, T]$ is $\sum_{k=1}^{M} k\theta_k$. It follows that the probability P_k that the i-th packet A_i is served in a busy period B_k can be obtained as follows

$$P_k = \lim_{T\to\infty} \frac{k\theta_k}{\sum_{k=1}^{M} k\theta_k} = \lim_{T\to\infty} \frac{k\times\frac{\theta_k}{\sum_{k=0}^{M}\theta_k}}{\sum_{k=1}^{M} k\times\frac{\theta_k}{\sum_{k=0}^{M}\theta_k}}$$
$$= \frac{kb_k}{\sum_{k=1}^{M} kb_k} = \frac{kb_k}{\overline{K}}.$$

■

Lemma 2. *The mean number of packets served ahead of the i-th packet A_i in the same busy period is given by*

$$E\left[\Delta_i\right] = \frac{\overline{K^2} - \overline{K}}{2\overline{K}}. \tag{2}$$

Proof: Conditioning on the event that packet A_i is served in a busy period B_k, we have

$$E\left[\Delta_i\right] = \sum_{k=1}^{M} E\left[\Delta_i | A_i \textit{ is served in a } B_k\right] P_k$$
$$= \sum_{k=1}^{M} \frac{0+1+\cdots+(k-1)}{k}\frac{kb_k}{\overline{K}}$$
$$= \sum_{k=1}^{M} \frac{\left(k^2-k\right)b_k}{2\overline{K}} = \frac{\overline{K^2}-\overline{K}}{2\overline{K}}.$$

■

As FIGURE 3 shows, it may take the server several busy periods to clear all packets waiting in the buffer ahead of packet A_i. Since the server can only transmit up to M packets in each busy period, before the starting of service, the waiting time of packet A_i includes the following components:

(1) The time to complete current service or current vacation. When packet A_i arrives, the residual time, either residual service time or residual vacation time, seen by A_i is denoted by R_i.
(2) The service times of all N_i packets found waiting in the buffer when A_i arrives.
(3) Besides residual vacation time, the duration of the whole vacation times experienced by A_i before the starting of service is denoted by Y_i.

It follows from the similar argument given in [21], we have

$$\overline{W} = \overline{R} + N_Q\overline{X} + \overline{Y}, \tag{3}$$

and

$$\overline{R} = E\left[R_i\right] = \frac{\lambda\overline{X^2}}{2} + \frac{(1-\rho)\overline{V^2}}{2\overline{V}}, \tag{4}$$

where $N_Q = E\left[N_i\right] = \lambda\overline{W}$ is the mean queue length, and $\rho = \lambda\overline{X}$ is the traffic load. The key to derive the mean waiting time (3) is the third term $\overline{Y} = E\left[Y_i\right]$, which is given in the proof of the following theorem.

Theorem 1. *The mean waiting time of an M/G/1 queue with vacations and limited-service discipline is given by*

$$\overline{W} = \frac{\frac{\lambda\overline{X^2}}{2} + \frac{(1-\rho)\overline{V^2}}{2\overline{V}} + \left[1 - \frac{(1+\rho)\left(\overline{K^2}-\overline{K}\right)}{2M\overline{K}} - \frac{\lambda\overline{V}}{M}\right]\overline{V}}{1-\rho-\frac{\lambda\overline{V}}{M}}. \tag{5}$$

Proof: Suppose the system is in state (n_i, m_i) when packet A_i arrives, meaning that the number of packets waiting in the buffer are n_i outside and m_i inside the gate. After A_i arrives, all m_i packets inside the gate are sent out during the first busy period, at the end of which the first M of the n_i packets enters the gate. Packet A_i enters the gate at the end of the $(\lfloor n_i/M \rfloor + 1)$-th busy period, and is sent out during the $(\lfloor n_i/M \rfloor + 2)$-th busy period. That is, the number of whole vacations that A_i has to experience before the starting of service is $\lfloor n_i/M \rfloor + 1$, where $\lfloor x \rfloor$ is the largest integer smaller than x.

For example, as FIGURE 3(a) shows, the state of system is $(n_i, m_i) = (4, 2)$ upon the arrival of A_i and M is three, thus A_i has to wait for $\lfloor n_i/M \rfloor + 1 = \lfloor 4/3 \rfloor + 1 = 2$ whole vacation times in the buffer before it can be transmitted. It's the same as that in FIGURE 3(b). Thus, we have

$$\overline{Y} = \left(1 + E\left[\left\lfloor \frac{n_i}{M} \right\rfloor\right]\right)\overline{V}. \tag{6}$$

In the $(\lfloor n_i/M \rfloor + 2)$-th busy period, the number of packets transmitted ahead of A_i is given by $\Delta_i = n_i - \lfloor \frac{n_i}{M} \rfloor M$. For example, as FIGURE 3(a) shows, packet A_i is the second packet served in the third busy period. Since $n_i = 4$ upon the arrival of A_i and $M = 3$, it follows that $\Delta_i = n_i - \lfloor \frac{n_i}{M} \rfloor M = 4 - 3 = 1$. However, in FIGURE 3(b), there is no packet transmitted before A_i in the third busy period since the system state is $(n_i, m_i) = (3, 3)$ when A_i arrives, and in this case $\Delta_i = n_i - \lfloor \frac{n_i}{M} \rfloor M = 3 - 3 = 0$. Therefore, by definition, we have

$$E\left[\left\lfloor \frac{n_i}{M} \right\rfloor\right] = \frac{E[n_i] - E[\Delta_i]}{M}. \tag{7}$$

Notice that the mean queue length N_Q is the sum of the mean number of packets waiting outside the gate $\overline{n} = E[n_i]$ and that waiting inside the gate $\overline{m} = E[m_i]$. It follows that

$$\overline{n} = E[n_i] = N_Q - \overline{m}. \tag{8}$$

Since the packet A_i is moved into the gate at the end of the $(\lfloor n_i/M \rfloor + 1)$-th busy period and served in the $(\lfloor n_i/M \rfloor + 2)$-th busy period, the waiting time of A_i inside the gate, denoted as W_{in}^i, includes the vacation time V between these two busy periods, and the total service time of Δ_i packets transmitted ahead of A_i in the $(\lfloor n_i/M \rfloor + 2)$-th busy period. Thus, the mean waiting time of A_i inside the gate is given by

$$\overline{W}_{in} = E\left[W_{in}^i\right] = \overline{V} + E[\Delta_i]\overline{X}.$$

From Little's Law and Lemma 2, we obtain the following mean number of packets waiting inside the gate:

$$\overline{m} = \lambda \overline{W}_{in} = \lambda \overline{V} + \frac{\rho\left(\overline{K^2} - \overline{K}\right)}{2\overline{K}}. \tag{9}$$

The theorem is established by combining (3)-(4) and (6)-(9). ■

Suppose the distribution of service time X is given. The evaluation of the mean waiting time (5) requires the first two moments of the vacation time V and the number of packets K transmitted in a busy period. Intuitively, they are related to each other because the random variable K depends on the number of arrivals during the vacation time V. Focusing on the application of the above theorem to EPONs, we will discuss the relationship between the first two moments of V and K in the next subsection.

C. MEAN WAITING TIME OF EPONS WITH LIMITED-SERVICE DISCIPLINE

In this subsection, we apply the result of Theorem 1 to calculate the mean waiting time of a limited-service EPON, where the rate of packet input to the network is λ_E packets/s and to each ONU is λ packets/s. The distribution of packet transmission time can be derived from Ethernet frame size distribution.

1) *Moments of Vacation Time V of An ONU*: An ONU is busy with probability $\rho = \lambda\overline{X}$ and idle with probability $1-\rho$. The mean busy period of an ONU is given by

$$\overline{B} = E[B] = \frac{\rho\overline{V}}{1-\rho}. \tag{10}$$

In an EPON with N ONUs where RTT is negligible, the vacation time of an ONU equals the TWs of other $(N-1)$ ONUs plus NG. According to our assumption A2, the TWs are i.i.d. random variables. By definition, we have

$$\overline{V} = (N-1)\overline{B} + NG = (N-1)\frac{\frac{\rho_E}{N}\overline{V}}{1-\frac{\rho_E}{N}} + NG,$$

where $\rho_E = \lambda_E\overline{X} = N\rho$ is the offered load to the EPON. After some reconfiguration, the first moment of vacation time V for an ONU is given by

$$\overline{V} = \frac{N-\rho_E}{1-\rho_E}G. \tag{11}$$

Similarly, the second moment of vacation time V for an ONU is defined as follows

$$\overline{V^2} = \overline{V}^2 + \sigma_V^2 = \overline{V}^2 + (N-1)\sigma_B^2, \tag{12}$$

where σ_V^2 and σ_B^2 are the variances of V and B, respectively. Recall that B_k is a busy period during which k packets are transmitted. It follows that $B_k = \sum_{i=1}^k X_i$, in which $X_1, X_2, \cdots, X_k$ are i.i.d. random variables. Let $X^*(\theta)$ and $B^*(\theta)$ be the Laplace-Stieltjes transforms of the probability density function (PDF) of service time X and busy period B, respectively. They are related as follows:

$$\begin{aligned} B^*(\theta) &= E\left[e^{-\theta B}\right] = E\left[E\left[e^{-\theta B}|B = B_k\right]\right] \\ &= \sum_{k=0}^{M} E\left[e^{-\theta B_k}\right] b_k = \sum_{k=0}^{M}\left(\prod_{i=1}^{k} E\left[e^{-\theta X_i}\right]\right) b_k \\ &= \sum_{k=0}^{M} \left[X^*(\theta)\right]^k b_k = F\left[X^*(\theta)\right], \end{aligned}$$

where $F(z) = \sum_{k=0}^M b_k z^k$ is the generating function of b_k. Therefore, the variance of busy period σ_B^2 can be obtained by

$$\begin{aligned} \sigma_B^2 &= B^{*\prime\prime}(0) - \left[-B^{*\prime}(0)\right]^2 \\ &= F''(1)[X^{*\prime}(0)]^2 + F'(1)X^{*\prime\prime}(0) - [F'(1)X^{*\prime}(0)]^2 \\ &= \left(\overline{K^2} - \overline{K}\right)\overline{X}^2 + \overline{K}\overline{X^2} - \left(\overline{K}\overline{X}\right)^2 \\ &= \overline{X}^2\left(\overline{K^2} - \overline{K}^2\right) + \overline{K}\left(\overline{X^2} - \overline{X}^2\right). \end{aligned} \tag{13}$$

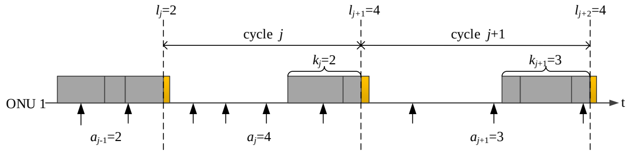

FIGURE 4: Upstream transmission process of an ONU, where $M = 3$.

Substituting (13) into (12), we obtain the following expression of $\overline{V^2}$:

$$\overline{V^2} = \overline{V}^2 + (N-1)\left[\overline{X}^2\left(\overline{K^2} - \overline{K}\right) + \overline{K}\left(\overline{X^2} - \overline{X}\right)\right]. \tag{14}$$

From (5), (11) and (14), we know that the mean waiting time of the EPON can now be determined by the first two moments $\overline{K}$ and $\overline{K^2}$ of the number of packets transmitted in a busy period.

2) *Moments of Number of Packets K Transmitted in A Busy Period*: The first moment of K can be easily derived from (10) and (11), and is given as follows:

$$\overline{K} = \frac{\overline{B}}{\overline{X}} = \frac{\lambda\overline{V}}{1-\rho} = \frac{\frac{\lambda_E}{N}\overline{V}}{1-\frac{\rho_E}{N}} = \frac{\lambda_E G}{1-\rho_E}. \tag{15}$$

The derivation of the second moment $\overline{K^2}$, however, has to resort to the discrete time Markov chain embedded in the epochs at the end of busy periods. As FIGURE 4 shows, the upstream transmission process of an ONU is a sequence of cycles. For example, at the end of cycle $j-1$ and the beginning of cycle j, the ONU reports its queue length, denoted as l_j, to the OLT. According to this report, the OLT determines the size of the TW in cycle j, denoted as k_j, as follows: $k_j = M$ if $l_j \geq M$, and $k_j = l_j$, if $l_j < M$. That is, the TW size k_j in cycle j is determined by the queue length l_j at the start of cycle j and is given as follows

$$k_j = l_j - (l_j - M)^+, \tag{16}$$

where $(l_j - M)^+ \triangleq \max\{l_j - M, 0\}$ is the number of reported packets that are not transmitted in cycle j.

Let $q_n = \lim_{j\to\infty} Pr\{l_j = n\}$. Recall that b_k is the probability that k packets are served in a busy period. According to (16), we have

$$b_k = \begin{cases} q_k, & k = 0, 1, \cdots, M-1 \\ 1 - \sum_{k=0}^{M-1} q_k, & k = M \end{cases} \tag{17}$$

Thus, the second moment of K can be obtained based on the distribution of the queue length q_n as follows

$$\overline{K^2} = \sum_{k=0}^{M} k^2 b_k = \sum_{k=0}^{M-1} k^2 q_k + M^2\left(1 - \sum_{k=0}^{M-1} q_k\right). \tag{18}$$

On the other hand, the queue length l_{j+1} at the start point of cycle $j+1$ is determined by the number of packets k_j transmitted during the busy period of cycle j and the number of arrivals a_j during cycle j. Thus, from (16), the queue length at the start point of each cycle satisfies the following Lindley's equation:

$$l_{j+1} = l_j - k_j + a_j = (l_j - M)^+ + a_j. \tag{19}$$

Let $h_n = \lim_{j\to\infty} Pr\{a_j = n\}$ be the probability that there are n arrivals during a cycle time C. We immediately derive the following equilibrium equation from (19):

$$q_n = \sum_{i=0}^{M-1} q_i h_n + \sum_{i=M}^{M+n} q_i h_{n+M-i},$$

from which we obtain the generating function of queue length:

$$Q(z) \triangleq \sum_{n=0}^{\infty} q_n z^n = \frac{\left[\sum_{i=0}^{M-1} q_i\left(z^M - z^i\right)\right] H(z)}{z^M - H(z)}, \tag{20}$$

where $H(z) \triangleq \sum_{n=0}^{\infty} h_n z^n$ is the generating function of h_n.

According to our assumption A3 that the packet arrival process of each ONU is a Poisson process, the distribution h_n is completely determined by the cycle time distribution. Let $c(t)$ be the PDF of the cycle time C. We have

$$H(z) = \sum_{n=0}^{\infty}\left[\int_0^{\infty}\frac{(\lambda t)^n}{n!}e^{-\lambda t}c(t)\,dt\right]z^n = C^*\left[\lambda(1-z)\right]. \tag{21}$$

A cycle consists of a vacation period and a busy period, which means that the cycle time is the sum of NG and the duration of the TWs of N ONUs, which are i.i.d. random variables according to our assumption A2. It follows that the distribution of cycle time is approximately a Gaussian distribution according to the central limit theorem [31]. The Laplace-Stieltjes transform of the cycle time distribution is given by

$$C^*(\theta) = exp\left[-\mu_C\theta + \frac{1}{2}\sigma_C^2\theta^2\right], \tag{22}$$

where the mean cycle time can be obtained from (11), and is given as follows:

$$\mu_C = \frac{\overline{V}}{1-\rho} = \frac{\frac{(N-\rho_E)G}{1-\rho_E}}{1-\frac{\rho_E}{N}} = \frac{NG}{1-\rho_E}, \tag{23}$$

while the variance of cycle time C is determined by the variance of busy period (13):

$$\sigma_C^2 = N\sigma_B^2 = N\left[\overline{X}^2\left(\overline{K^2} - \overline{K}^2\right) + \overline{K}\left(\overline{X^2} - \overline{X}^2\right)\right]. \tag{24}$$

We know that the second moment $\overline{K^2}$ in (18) is coupled to the queue length probability q_n, for $n = 0, 1, \cdots, M-1$. For a given $\overline{K^2}$, according to Rouche's theorem and Lagrange's theorem [20], we can first solve q_n $(n = 0, 1, \cdots, M-1)$ from (20)-(24), and then update $\overline{K^2}$ by substituting q_n into (18). Repeatedly applying this iterative procedure, we can obtain the value of $\overline{K^2}$ and then obtain the mean waiting time (5) of an ONU by combining (11), (14), (15), and $\overline{K^2}$. APPENDIX A gives the procedure that numerically calculates $\overline{K^2}$ and the mean waiting time. In the next section, we seek a systematic rule to select the optimum TW size for each ONU of the EPON that satisfies the practical operational requirements of limited-service EPONs.

IV. OPTIMUM TRANSMISSION WINDOW SIZE

In limited-service EPONs, each user signs a service level agreement (SLA) with the network operator to subscribe to a traffic rate, denoted by r^*, in units of Bytes/s or B/s. When the subscribed traffic rate r^* is specified, the maximal number of packets that the user can inject into the network per second, called the subscribed packet-arrival rate of the user and denoted by λ^*, is determined accordingly. Let R be the upstream transmission capacity of the EPON in units of B/s. Recall that the mean service time of packets is $\overline{X}$ second, thus the subscribed packet-arrival rate is

$$\lambda^* = \frac{r^*}{R\overline{X}}\, packets/s. \tag{25}$$

Let r be the input traffic rate of each ONU in units of B/s. An ONU is a disciplined user and its QoS should be guaranteed if its input traffic rate or the packet-arrival rate is in the subscribed region, that is $r \in [0, r^*]$ or $\lambda = \lambda_E/N \in [0, \lambda^*]$. Otherwise, it is a malicious user. In a homogenous EPON, an ONU is either a disciplined user or a malicious user. An EPON system is $regular$ if all the ONUs are disciplined users. In this section, we describe the methodology and procedure to select an optimum TW size M based on the analytical result in Section III, given the subscribed traffic rate r^* or the subscribed packet-arrival rate λ^* of each ONU.

As previously stated, the purpose of limiting the TW size is to guarantee the QoS of disciplined users and to penalize malicious users. The TW size M limits the maximum number of packets that can be served in a busy period. Ideally, a proper TW size M ensures that all packets arrived at a disciplined ONU during a cycle time can be completely served in the next busy period. To achieve this goal, the probability that the queue length at the beginning of a cycle exceeds the limit M should be kept very small. Thus, the criterion for the selection of TW size M is given by

$$Pr\{l \geq M\} = \lim_{j\to\infty} Pr\{l_j \geq M\} \leq \varepsilon, \tag{26}$$

for some positive $\varepsilon \ll 1$, when $\lambda = \lambda_E/N \in [0, \lambda^*]$. In the following, we show an optimum TW size M that satisfies criterion (26) can be selected by using the Chernoff bound of queue length.

A. CHERNOFF BOUND OF QUEUE LENGTH

The Chernoff bound of the tail distribution of queue length l at the beginning of a cycle is given as follows [32]:

$$Pr\{l \geq \mu_l + t\} = Pr\{z^l \geq z^{\mu_l+t}\} \leq \frac{E\left[z^l\right]}{z^{\mu_l+t}}, \tag{27}$$

for any $z > 1$, where $E[z^l] = Q(z)$ is the generating function of the queue length distribution and $\mu_l = E[l]$ is the mean queue length.

Suppose all the ONUs are disciplined users with packet-arrival rate $\lambda = \lambda_E/N \in [0, \lambda^*]$, and the TW size M satisfies criterion (26), then each time the queue length reported by an ONU should be typically smaller than M with a high probability $1 - \varepsilon$. It follows that equations (16) and (19) will respectively degenerate to the following approximate equations:

$$l_j \approx k_j, \tag{28}$$

$$l_{j+1} \approx a_j, \tag{29}$$

which implies that the following generating functions of l_j, k_j and a_j are approximately equal:

$$Q(z) \approx F(z) \approx H(z). \tag{30}$$

Thus, according to (21)-(24), we have

$$\begin{aligned} Q(z) \approx H(z) =& exp\left[-\lambda\mu_C(1-z) + \frac{1}{2}\lambda^2\sigma_C^2(1-z)^2\right] \\ =& exp\left\{-\frac{\lambda_E G\,(1-z)}{1-\rho_E} + \left[\overline{X}^2\left(\overline{K^2}-\overline{K}^2\right)\right.\right. \\ &\left.\left. + \overline{K}\left(\overline{X^2}-\overline{X}^2\right)\right]\frac{\lambda_E^2(1-z)^2}{2N}\right\}. \end{aligned} \tag{31}$$

In this equation, the second moment of the number of packets served in each busy period can be obtained by

$$\overline{K^2} = F''(1) + F'(1) \approx H''(1) + H'(1). \tag{32}$$

Substituting (31) into (32), we have

$$\begin{aligned} \overline{K^2} =& \left(\frac{\lambda_E G}{1-\rho_E}\right)^2 + \frac{\rho_E^2}{N}\left[\overline{K^2} - \left(\frac{\lambda_E G}{1-\rho_E}\right)^2\right] \\ &+ \frac{\lambda_E^3 G}{N\,(1-\rho_E)}\left(\overline{X^2}-\overline{X}^2\right) + \frac{\lambda_E G}{1-\rho_E}, \end{aligned}$$

which yields

$$\overline{K^2} = \left(\frac{\lambda_E G}{1-\rho_E}\right)^2 + \frac{\frac{\lambda_E^3 G}{N(1-\rho_E)}\left(\overline{X^2}-\overline{X}^2\right) + \frac{\lambda_E G}{1-\rho_E}}{1-\frac{\rho_E^2}{N}}. \tag{33}$$

Substituting (33) into (31), we obtain $Q(z)$ in the regular case as follows:

$$Q(z) \approx exp\left[-\frac{\lambda_E G}{1-\rho_E}(1-z) + \frac{\lambda_E^3 G\overline{X^2}}{2\,(1-\rho_E)\,(N-\rho_E^2)}(1-z)^2\right]. \tag{34}$$

The mean and variance of queue length l are respectively given as follows:

$$\mu_l = Q'\,(1) = \lambda\mu_C = \frac{\lambda_E G}{1-\rho_E}, \tag{35}$$

and

$$\begin{aligned} \sigma_l^2 &= Q''(1) + Q'(1) - \left[Q'(1)\right]^2 = \lambda^2\sigma_C^2 + \lambda\mu_C \\ &= \frac{\lambda_E^3 G\overline{X^2}}{(1-\rho_E)\,(N-\rho_E^2)} + \frac{\lambda_E G}{1-\rho_E}. \end{aligned} \tag{36}$$

It follows from the first equation of generating function of queue length in (31), the Chernoff bound (27) is given by

$$Pr\{l \geq \mu_l + t\} \leq$$
$$exp\left[-(\mu_l + t)\log z + \lambda\mu_C(z-1) + \frac{1}{2}\lambda^2\sigma_C^2(z-1)^2\right],$$

for any $z > 1$. Substituting $t = M - \mu_l$ into the above Chernoff bound, then criterion (26) can be fulfilled if M is the smallest integer that satisfies the following inequality:

$$Pr\{l \geq M\} \leq$$
$$\inf_{z>1}\left\{exp\left[-M\log z + \lambda\mu_C(z-1) + \frac{1}{2}\lambda^2\sigma_C^2(z-1)^2\right]\right\} \leq \varepsilon, \tag{37}$$

where $\lambda = \lambda_E/N \in [0, \lambda^*]$. We discuss the procedure to find the optimum TW size M^* that satisfies (37) in the next subsection.

B. PROCEDURE OF DETERMINING OPTIMUM TW SIZE

Solving the optimum TW size M^* from (37) involves a complicated transcendental equation, therefore it can only be solved numerically. To initialize the computation procedure, we provide a lower bound M_1 and an upper bound M_2 of M^* in the following theorem.

Theorem 2. *The optimum TW size M^* that satisfies (37) is bounded by*

$$\left\lceil \mu_l + \lambda\sigma_C\sqrt{2\alpha} \right\rceil = M_1 \leq M^* \leq M_2 = \left\lceil \mu_l + \alpha + \sqrt{\alpha^2 + 2\alpha\sigma_l^2} \right\rceil \tag{38}$$

where $\alpha = \log\varepsilon^{-1}$ and $\lambda = \lambda_E/N \in [0, \lambda^*]$.

■

APPENDIX B gives the proof of the above theorem. An accurate approximation of the optimum TW size M^* can be derived from the upper deviation inequality of normal random variables. We know that cycle time C approaches a normal random variable $\mathcal{N}(\mu_C, \sigma_C^2)$ when N is large. The relation (29) indicates that the queue length at the beginning of each cycle is approximately equal to the number of arrivals during a cycle time C, or $l \sim \lambda C$. As expected, the mean queue length μ_l given by (35) is the product of packet-arrival rate λ and mean cycle time μ_C. It is also interesting to note that the variance of the queue length σ_l^2 given by (36) is the sum of $\lambda^2\sigma_C^2$ and the variance of a Poisson random variable with parameter μ_l. Thus, the queue length l can be approximated by a normal random variable $\mathcal{N}(\mu_l, \sigma_l^2)$ that is "discretized" by a Poisson process with rate λ. That is, we adopt the following approximation of queue length distribution:

$$q_n \cong q'_n = \frac{1}{\sqrt{2\pi}\sigma_l}\int_{n-\frac{1}{2}}^{n+\frac{1}{2}} e^{-\frac{(x-\mu_l)^2}{2\sigma_l^2}}dx. \tag{39}$$

In this paper, we consider a typical FTTH network in which the split ratio is up to 1:64 and the service capacity of the upstream link is 10 Gb/s, which is equivalently 1.25 GB/s or 1250 MB/s [29], [30]. TABLE 1 lists the other parameters. As FIGURE 5 illustrates, the bigger the gap between these two distributions, the smaller the probability q_n, where q_n is obtained through the inverse transform of $Q(z)$ in (34).

It is well-known that any normal random variable $X \sim \mathcal{N}(\mu, \sigma^2)$ satisfies the following upper deviation inequality [33]:

$$Pr\{X \geq \mu + t\} \leq exp\left[-\frac{t^2}{2\sigma^2}\right],$$

for $t \geq 0$. Since the distribution of queue length l is close to that of the normal random variable $\mathcal{N}(\mu_l, \sigma_l^2)$, from the above inequality, the optimum TW size M^* can be estimated by the smallest integer $\hat{M}$ that satisfies the following relation:

$$Pr\{l \geq \hat{M}\} \leq exp\left[-\frac{\left(\hat{M} - \mu_l\right)^2}{2\sigma_l^2}\right] \leq \varepsilon,$$

Let $\lambda_E = N\lambda^* = \frac{Nr^*}{R\overline{X}}$, substituting (35) and (36) into above inequality, $\hat{M}$ can be explicitly given by the following equation (40).

The following inequality immediately follows from (36) and (40):

$$\left\lceil \mu_l + \lambda\sigma_C\sqrt{2\alpha} \right\rceil \leq \hat{M} \leq \left\lceil \mu_l + \alpha + \sqrt{\alpha^2 + 2\alpha\sigma_l^2} \right\rceil.$$

That is, the approximation $\hat{M}$ of the optimum TW size M^* also lies between the two bounds M_1 and M_2.

$$\begin{aligned}
\hat{M} = \left\lceil \mu_l + \sigma_l\sqrt{2\alpha} \right\rceil &= \left\lceil \frac{\lambda_E G}{1-\rho_E} + \sqrt{2\alpha\left[\frac{\lambda_E^3 G\overline{X^2}}{(1-\rho_E)(N-\rho_E^2)} + \frac{\lambda_E G}{1-\rho_E}\right]} \right\rceil \\
&= \left\lceil \frac{N\lambda^* G}{1-N\lambda^*\overline{X}} + \sqrt{2\log\varepsilon^{-1}\left[\frac{N^2\lambda^{*3}G\overline{X^2}}{(1-N\lambda^*\overline{X})\left(1-N\lambda^{*2}\overline{X}^2\right)} + \frac{N\lambda^* G}{1-N\lambda^*\overline{X}}\right]} \right\rceil \\
&= \left\lceil \frac{Nr^* G}{R\overline{X} - Nr^*\overline{X}} + \sqrt{2\log\varepsilon^{-1}\left[\frac{N^2 r^{*3}G\overline{X^2}}{(R-Nr^*)\left(R^2-Nr^{*2}\right)\overline{X}^3} + \frac{Nr^* G}{R\overline{X} - Nr^*\overline{X}}\right]} \right\rceil
\end{aligned} \tag{40}$$

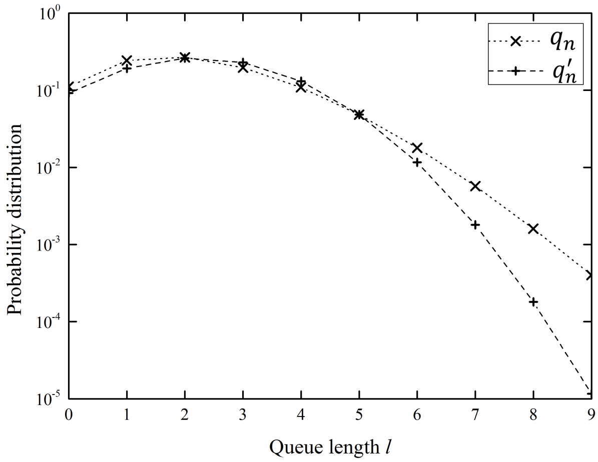

FIGURE 5: Probability distribution q_n and q'_n with $\lambda = \frac{r}{R\overline{X}} = 1.6 \times 10^4$ packets/s.

As previously mentioned, the optimum TW size M^* that satisfies the inequality (37) can only be solved numerically from the following equation:

$$f(t, z^*) = exp\left[-(\mu_l + t)\log z^* + \lambda\mu_C(z^* - 1) + \frac{1}{2}\lambda^2\sigma_C^2(z^* - 1)^2\right] = \varepsilon,$$

where z^* is obtained from the proof of Theorem 2 in APPENDIX B and is given as follows:

$$z^* = \frac{\sqrt{(\lambda\mu_C - \lambda^2\sigma_C^2)^2 + 4(\mu_l + t)\lambda^2\sigma_C^2} - (\lambda\mu_C - \lambda^2\sigma_C^2)}{2\lambda^2\sigma_C^2}. \quad (41)$$

The following procedure is used to solve the optimum TW size M^* that satisfies the inequality (37).

Step 1: $\lambda = \lambda^* = r^*/R\overline{X}, M = \hat{M}, low = M_1, up = M_2$;
Step 2: $t = M - \mu_l$, calculate z^* by (41);
Step 3: If $f(t, z^*) > \varepsilon$, $low = M$; else $up = M$;
/* If $f(t, z^*)$ is too large, we update the lower bound of searching region to decrease $f(t, z^*)$, otherwise we update the upper bound. */
Step 4: If $\lceil low \rceil < \lceil up \rceil$, $M = (low + up)/2$, go to Step 2;
Step 5: $M^* = \lceil low \rceil = \lceil up \rceil$, output M^*.

In the practical operation of EPON, the parameter ε can be selected from the region $[0.001, 0.1]$, which implies that the buffered packets of an ONU can be emptied with a probability $1-\varepsilon$ between 0.9 to 0.999 at the end of every busy period. A too large ε causes a too small M that impairs the delay performance of disciplined users. On the other hand, a too small ε causes a too large M that cannot effectively suppress the capture effect.

FIGURE 6(a) and 6(b) respectively illustrate that the optimum TW size M^*, its lower bound M_1, upper bound M_2, and approximation $\hat{M}$, vary with the tail bound ε and the subscribed traffic rate r^* of each ONU. In these figures, we find that both approximate and optimum TW sizes, $\hat{M}$ and

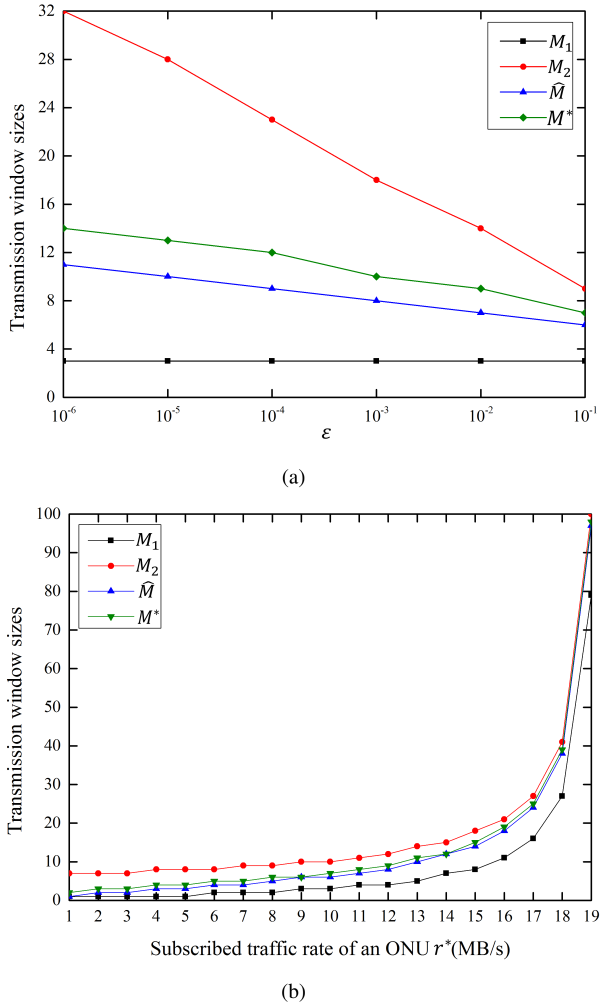

FIGURE 6: TW sizes vary with tail bound ε and subscribed traffic rate r^* respectively: (a) TW sizes v.s. ε where $r^* = 10$ MB/s and (b) TW sizes v.s. r^* where $\varepsilon = 0.05$.

M^*, are always bounded between M_1 and M_2. Moreover, the approximation $\hat{M}$ is uniformly smaller than M^*, which can be readily seen from the distributions illustrated in FIGURE 5. The convergence rate of normal distribution q'_n is faster than that of q_n, thus a smaller TW size is needed to achieve the same probability of tail distribution. In spite of that, as FIGURE 6(a) shows, the difference between $\hat{M}$ and M^* is very small in the region $\varepsilon \in [0.001, 0.1]$ of our interest in practice. Besides, the selection of TW size is very sensitive to the traffic rate a user subscribes to, as illustrated in FIGURE 6(b). With the growth of r^*, the TW size also increases greatly.

The tail distribution of queue length, $\sum_{n=M}^{\infty} q_n$, and that of approximation, $\sum_{n=M}^{\infty} q'_n$, are plotted in FIGURE 7 where each ONU inputs traffic with the rate of 10 MB/s. In our interested region $\varepsilon \in [0.001, 0.1]$, the difference between the TW sizes selected by tail distributions of q_n and q'_n is quite small. When their gap is large, such as in the area

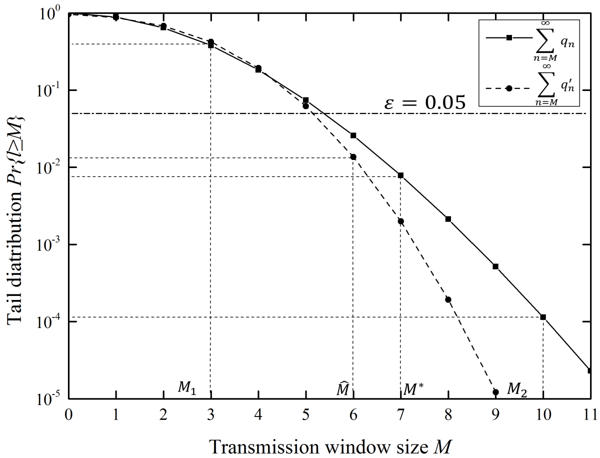

FIGURE 7: Tail distributions of q_n and q_n' with $r = 10$ MB/s.

$\varepsilon \in [10^{-5}, 10^{-4}]$, ε would be extremely small and far below the region of our interest in the practical operation of EPON.

For a fixed $\varepsilon = 0.05$, as FIGURE 7 shows, despite that $\hat{M} < M^*$, the probability $Pr\{l \geq \hat{M}\}$ is still below ε, which means the approximation $\hat{M}$ also satisfies criterion (26). If the TW size is set to equal the lower bound M_1, criterion $Pr\{l \geq M\} \leq \varepsilon$ could be violated and packets may experience longer delay than expected. On the other hand, if the TW size is set to equal the upper bound M_2, the criterion can be easily satisfied because $Pr\{l \geq M_2\}$ is negligible in comparison with ε. However, the upper bound M_2 would be too large to be an effective constraint on malicious users. As a compromise, the approximation $\hat{M}$ can serve as a practical TW size for EPONs.

C. STABILITY AND DELAY PERFORMANCE OF LIMITED-SERVICE EPON

In this subsection, we study the delay performance of disciplined ONUs in a limited-service EPON with the TW size limit M given by (40). The gated-service discipline is a special case of the limited-service discipline with infinite TW size, thus the mean waiting time in gated service is the lower bound of that in limited service.

The EPON system with gated service is stable if the offered load $\rho = \lambda \overline{X} = r/R$ of each ONU is less than $1/N$, i.e., $r < R/N$, which guarantees that input packets will be transmitted steadily and their mean waiting time, or mean queue length, is bounded. However, a bounded mean queue length is not sufficient to guarantee that a regular EPON with limited service is stable due to the limitation of TW size M. From the mean queue length formula (35) of an ONU, the stable condition of the limited service EPON is given by

$$\mu_l = \lambda \mu_C = \frac{r}{R\overline{X}} \cdot \frac{NG}{1 - \rho_E} < M, \tag{42}$$

where $\rho_E = N\rho = Nr/R$. After some algebraic manipulation, a stable input traffic rate r should be bounded by $\hat{r}$ that is defined as follows:

$$r < \hat{r} = \frac{MR\overline{X}}{N\left(M\overline{X} + G\right)} \tag{43}$$

It's evident that when $M \to \infty$, the saturated input traffic rate $\hat{r} \to R/N$. Furthermore, the TW size limit M is selected based on criterion (26), which guarantees a very small probability ε that the queue length will exceed limit M. This is a much more stringent condition than the stable condition (42). Therefore, in a regular EPON with limited service, disciplined ONUs with input traffic rate in the region $r \in [0, r^*]$ must be all stable, which implies $r^* \leq \hat{r}$.

According to the conditions described above, the performance of an ONU in a limited-service EPON can be characterized in the following three traffic regions:

1) Subscripted region $r \in [0, r^*]$. In the subscripted region, the QoS of each ONU in terms of mean delay is guaranteed by the SLA signed with the network operator.
2) Overloaded region $r \in (r^*, \hat{r})$. If an ONU inputs the packets with the rate higher than its subscripted rate and in the overloaded region, the mean delay is impaired by the limit of the maximum TW size M, but it is still bounded. This region provides an adjustment period for the ONU to decrease its input traffic rate when the user experiences a larger than expected delay.
3) Saturated region $r \in \left[\hat{r}, \frac{R}{N}\right)$. In the saturated region, the arrival rate is too high for the OLT to handle. The ONU is unstable when the queue length outside the gate of the buffer becomes unbounded.

In the subscripted region $r \in [0, r^*]$ or in the overloaded region $r \in (r^*, \hat{r})$, the mean waiting time of an ONU can be calculated by the procedure described in APPENDIX A. In the saturated region $r \in \left[\hat{r}, \frac{R}{N}\right)$, the mean waiting time is unbounded.

TABLE 1: Simulation Parameters

Parameter	Value
Number of ONUs N	64
Upstream link capacity R	10 Gb/s (i.e., 1.25 GB/s or 1250 MB/s)
Packet size distribution	64 Bytes (47%), 300 Bytes (5%), 594 Bytes (15%), 1300 Bytes (5%), 1518 Bytes (28%) [8]
Guard time	1 μs

The analytical results of mean waiting times in these traffic regions are verified by simulations, where the simulation parameters are listed in TABLE 1. According to the packet size distribution, we can easily obtain the first and second moment of service time as $\overline{X} = 0.5\mu s$ and $\overline{X^2} = 0.5\mu s^2$. The guard time plus the transmission time of a 64-Byte REPORT message constitute a constant interval $G = 1.0512\mu s$. The service capacity of the EPON is 1250 MB/s, evenly divided into 19.53125 MB/s for each of 64 ONUs. We study a scenario, where each user subscribes to a traffic rate of 8 MB/s. According to formula (40), for a fixed $\varepsilon = 0.05$, we should set the maximum TW size M to 5.

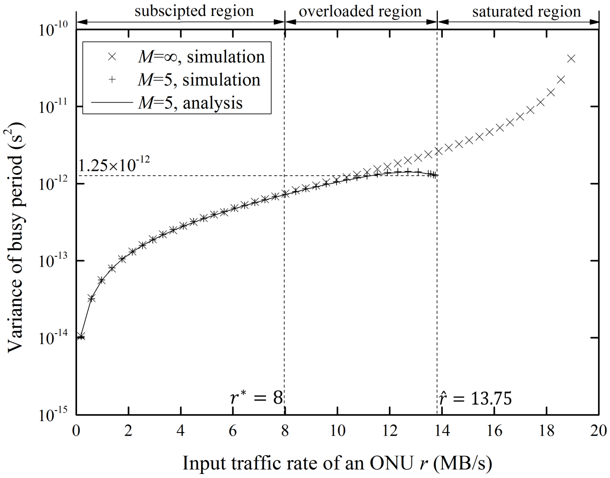

FIGURE 8: Variance of busy periods versus input traffic rate of an ONU where $\varepsilon = 0.05$ and $r^* = 8$ MB/s.

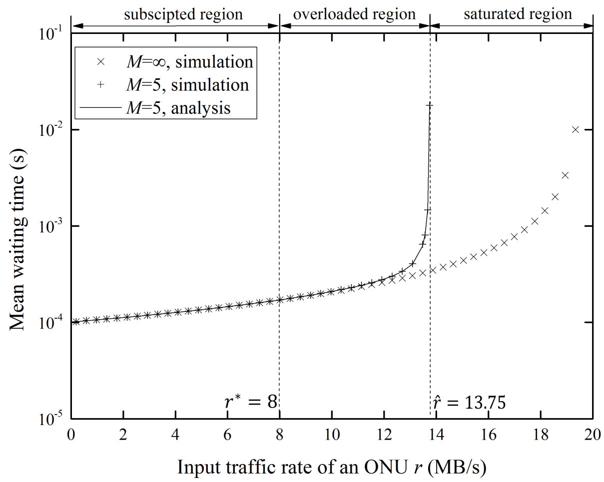

FIGURE 9: Mean waiting time versus input traffic rate of an ONU where $\varepsilon = 0.05$ and $r^* = 8$ MB/s.

FIGURE 8 illustrates the variance of busy periods for each ONU. It is evident that the analytical result given in APPENDIX A is consistent with the simulation result, which validates the accuracy of our analysis in Section III. If we adopt the gated-service discipline (i.e., M is infinite), FIGURE 8 shows that the variance of busy periods monotonically increases with input traffic rate up to infinity. However, with limited-service discipline (i.e., M is finite), the variance of busy periods approaches to $M(\overline{X^2} - \overline{X}^2)$ according to (13), because each ONU transmits a constant number of M packets in each busy period when the traffic rate is high.

In the subscripted region, as FIGURE 9 shows, the disciplined users in the limited-service EPON experience the same mean waiting time as that in the gated-service EPON. This desirable property is due to the criterion $Pr\{l \geq M\} \leq \varepsilon$ of selecting the TW size , which is sufficiently large to empty the buffered packets almost in every busy period.

In the overloaded region, the ONU will suffer a larger mean delay than expected, which serves as a precaution measure for the ONU to reduce the loading back to the subscripted region. If the ONU continues increasing the input traffic rate to the saturated region, its mean delay tends to infinity, and the service is collapsed to prevent its malicious behavior from impacting the QoS of other disciplined users.

D. IMPACT OF RTT

In the practical EPON, there is a RTT, denoted by T_{RTT}, between the OLT and ONUs, as we mentioned in Section III. In this subsection, we discuss the influence of the RTT on the mean waiting time and the TW-size selection. In general, the impact of the RTT on the mean waiting time and the TW-size selection is significant only when the traffic load of the network is small.

1)*Impact on the Mean Waiting Time*:The RTT between the ONUs and the OLT has remarkable impact on the mean cycle time, and thus the mean waiting time when the traffic load is low. As Section III describes, an ONU takes a vacation after it sends an REPORT to the OLT at the end of a busy period. The vacation time of an ONU terminates when all other $N-1$ ONUs have finished transmission and this ONU receives the GATE message from the OLT. Recall that the ONU can receive the GATE message at the earliest after the RTT since this ONU sends the REPORT. Thus, the vacation time is the maximum of T_{RTT} and the sum of the TWs of other $N-1$ ONUs plus NG. For example, the vacation time of ONU 1 is

$$V = \max\left\{T_{RTT}, NG + \sum_{j=2}^{N} B_j\right\}, \tag{44}$$

where B_j is the busy period of the j-th ONU. It is clear that

$$V = T_{RTT}, \tag{45}$$

if and only if

$$T_{RTT} \geq NG + \sum_{j=2}^{N} B_j.$$

Under this situation, we have

$$T_{RTT} \geq E\left[NG + \sum_{j=2}^{N} B_j\right] = NG + \frac{(N-1)\rho\overline{V}}{1-\rho} = NG + \frac{(N-1)\rho T_{RTT}}{1-\rho},$$

which implies

$$\rho = \frac{r}{R} \leq \frac{T_{RTT} - NG}{NT_{RTT} - NG},$$

or

$$r \leq \left(\frac{T_{RTT} - NG}{NT_{RTT} - NG}\right) R. \tag{46}$$

In the following, we discuss the mean waiting time by dividing the input traffic rate of an ONU into two regions with respect to $r = \left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R$.

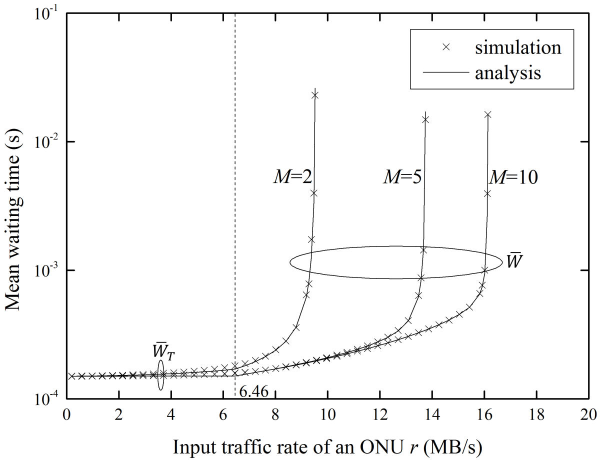

FIGURE 10: Mean waiting time versus with input traffic rate of an ONU where $N = 64$ and $T_{RTT} = 100\mu s$.

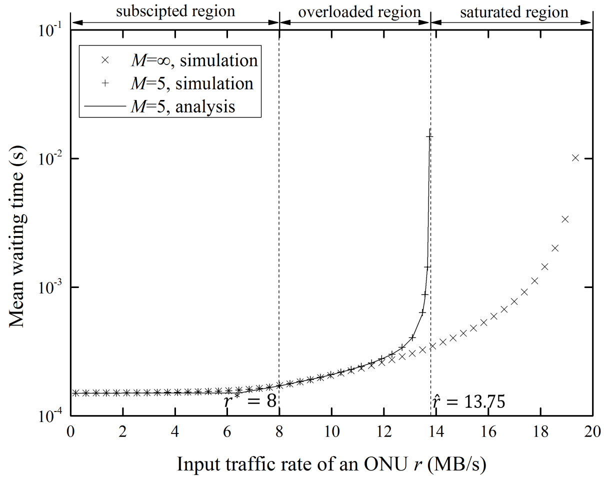

FIGURE 11: Mean waiting time of an ONU under that case where $r^* = 8$ MB/s and $T_{RTT} = 100\mu s$.

On one hand, if the input traffic rate r of each ONU is smaller than $\left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R$ such that $T_{RTT} > NG + \sum_{j=2}^{N} B_j$ with high probability, the vacation time is approximately a constant $V \approx T_{RTT}$, which means

$$\overline{V} = T_{RTT}, \tag{47}$$

$$\overline{V^2} = T_{RTT}^2. \tag{48}$$

In this case, the cycle time is also approximately a constant $C = B + V \approx T_{RTT}$, since the busy period is much smaller than the vacation time V. In this case, from equation (5), we obtain the following mean waiting time

$$\overline{W}_T = \frac{\frac{\lambda\overline{X^2}}{2} + \frac{(1-\rho)T_{RTT}}{2}}{1-\rho-\frac{\lambda T_{RTT}}{M}} + \frac{\left[1-\frac{(1+\rho)(\overline{K^2}-\overline{K})}{2M\overline{K}} - \frac{\lambda T_{RTT}}{M}\right] T_{RTT}}{1-\rho-\frac{\lambda T_{RTT}}{M}}, \tag{49}$$

where the first moment of the number of packets served in a busy period is given by

$$\overline{K} = \frac{\lambda\overline{V}}{1-\rho} \approx \frac{\lambda T_{RTT}}{1-\rho},$$

and the second moment $\overline{K^2}$ can be calculated by using the method given in APPENDIX A.

On the other hand, if the input traffic rate r of each ONU is larger than $\left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R$ such that $T_{RTT} < NG + \sum_{j=2}^{N} B_j$ with high probability, the vacation time is $V \approx NG + \sum_{j=2}^{N} B_j$. In this case, the mean waiting time, denoted by $\overline{W}$, can be obtained by the procedure described at the end of Section III.

In summary, taking the RTT into consideration, the mean waiting time $\overline{W}_{RTT}$ is approximately given by:

$$\overline{W}_{RTT} \approx \begin{cases} \overline{W}_T, & r < \left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R \\ \overline{W}, & r > \left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R. \end{cases} \tag{50}$$

As an example, we consider a 64-ONU FTTH, in which the distance between the OLT and ONUs is about 10 km such that the RTT is 100 μs [34]. The other parameters are the same as that listed in TABLE 1. As FIGURE 10 shows, the result of (50) agrees with the simulation under different values of M. In particular, the mean waiting time $\overline{W}_{RTT}$ is equal to $\overline{W}_T$ when

$$r < \left(\frac{T_{RTT}-NG}{NT_RTT-NG}\right) R = \frac{100-64\times 1.0512}{64\times 100 - 64\times 1.0512} \times 1250 = 6.46MB/s,$$

and it is equal to $\overline{W}$ when input traffic rate $r > 6.46$ MB/s. Please note that the upstream capacity that can be provided to each ONU is about 20 MB/s. This indicates that the RTT has a significant impact on the mean waiting time only in a small region of the input traffic rate in a typical FTTH network with 64 ONUs.

2)*Impact on the TW-size Selection Rule*: The RTT also has a remarkable impact on the TW-size selection only when the subscribed traffic rate r^* is small. Recall that, we select the TW size M according to the Chernoff bound of the queue length at the beginning of a cycle. As we described above, when r^* is small, e.g., $r^* < \left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R$, the RTT has a profound influence on the cycle time and the queue length at the beginning of a cycle. Thus, the selection rule (40) does not work when r^* is small. On the other hand, if $r^* > \left(\frac{T_{RTT}-NG}{NT_{RTT}-NG}\right) R$, the cycle time almost does not depend on the RTT and it is approximately equal to $NG + \sum_{j=2}^{N} B_j$. In this case, we can use (40) to determine M. To verify this point, we reconsider the example in FIGURE 11, under the scenario where the subscribed traffic rate of each ONU is $r^* = 8$ MB/s. Since r^* is larger than 6.46 MB/s, we select $M = 5$ using (40). As FIGURE 11 shows, the mean waiting time of the limited-service EPON is the same as that of the gated-service EPON when the input traffic rate $r < r^*$, while it is larger than that of the gated-service EPON when $r > r^*$. Once the input traffic rate r exceeds the saturated traffic rate $\hat{r}$, the mean waiting time of the limited-service EPON skyrockets to infinity immediately. We observed the same phenomenon in FIGURE 9, in which the RTT is ignored.

V. CONCLUSION

In this paper, we study the problem of the optimum TW-size selection for limited-service EPON. We first model each ONU as an M/G/1 queue with vacations and limited-service discipline. We extend the traditional geometric approach to obtain the mean waiting time formula. Based on the mean waiting time and the Chernoff bound of the queue length, we provide a selection rule of the optimum TW size. Our results show that the limited-service EPON with our TW-size selection rule can ensure the QoS of the discipline users while punishing the malicious users in terms of mean waiting time.

Currently, we focus our study on EPONs with statistically identical ONUs and almost the same RTT between ONUs and the OLT. Moreover, we only consider the traditional EPON with one-thread DBA algorithm in our analysis. In the future, it would be interesting to extend our model to analyze cases that are more sophisticated. These cases include the EPON with ONUs that are not statistically identical, the EPON in which the RTTs of various ONUs are different, or the EPON that employs multi-thread DBA algorithms for very large RTTs [35]–[37].

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APPENDIX A ITERATIVE PROCEDURE OF CALCULATING MEAN WAITING TIME $\overline{W}$ AND VARIANCE OF BUSY PERIODS σ_B^2

As analyzed in Section III, it is critical to obtain the second moment of the number of packets served in a busy period $\overline{K^2}$ when calculating the mean waiting time and variance of busy periods. However, the value of $\overline{K^2}$ and that of distribution q_n $(n = 0, 1, \cdots, M-1)$ depend on each other, and we can only solve them numerically.

According to Rouche's theorem, the denominator of (20) has M zeros inside and on $|z| = 1$, one of them is $z = 1$. Then by Lagrange's theorem [20], the other $(M-1)$ zeros inside $|z| = 1$ are given by

$$z_m = \sum_{n=1}^{\infty} \frac{e^{2\pi mni/M}}{n!} \frac{d^{n-1}}{dz^{n-1}} \Big[H(z)\Big]^{n/M}\Bigg|_{z=0}, \tag{51}$$

for $m = 1, 2, \cdots, M-1$. Since $Q(z)$ is analytic in $|z| \le 1$, the numerator of (20) must also be zero at $z = z_m$. Therefore, q_n $(n = 0, 1, \cdots, M-1)$ satisfy the following $(M-1)$ linear equations:

$$\sum_{n=0}^{M-1} q_n \left(z_m^M - z_m^n\right) = 0, \quad m = 1, 2, \cdots, M-1. \tag{52}$$

Another equation is given as follows by the condition $Q(1) = 1$:

$$\sum_{n=0}^{M-1} q_n (M-n) = M - \lambda\mu_C = M - \frac{\lambda_E G}{1-\rho_E}. \tag{53}$$

Thus, if we know the expression of $H(z)$, we can solve q_n $(n = 0, 1, \cdots, M-1)$ by combining (51)-(53), then obtain $\overline{K^2}$ based on (18), which is

$$\overline{K^2} = \sum_{n=0}^{M-1} n^2 q_n + M^2 \left(1 - \sum_{n=0}^{M-1} q_n\right).$$

However, the expression of $H(z)$ is instead dependent on $\overline{K^2}$. Therefore, given a calculation accuracy δ, we can numerically solve $\overline{K^2}$ through the following iteration procedure:

Step 1: $\overline{K^2} = 0$;
Step 2: Calculate $H(z)$ by combining (15), (21)-(24);
Step 3: Solve $z_m, m = 1, 2, \cdots, M-1$ by (51);
Step 4: Solve $q_n, n = 0, 1, \cdots, M-1$ by combining (52) and (53);
Step 5: If $\left|\sum_{n=0}^{M-1} n^2 q_n + M^2 \left(1 - \sum_{n=0}^{M-1} q_n\right) - \overline{K^2}\right| > \delta$, $\overline{K^2} = \sum_{n=0}^{M-1} n^2 q_n + M^2 \left(1 - \sum_{n=0}^{M-1} q_n\right)$, go to Step 2;
Step 6: Output $\overline{K^2}$.

Then, we can easily obtain the variance of busy periods for an ONU by substituting $\overline{K^2}$ and (15) into (13), and the mean waiting time by combining (5), (11), (14), (15) and $\overline{K^2}$.

APPENDIX B PROOF OF THEOREM 2

Define the following function:

$$f(t,z) = exp\Big[-(\mu_l + t)\log z + \lambda\mu_C(z-1) + \frac{1}{2}\lambda^2\sigma_C^2(z-1)^2\Big], \tag{54}$$

where $z > 1$ and $t \ge 0$. We know that the following inequality holds for all $x \ge 0$:

$$-x \le -\log(1+x) \le -\left(x - \frac{1}{2}x^2\right). \tag{55}$$

Let $x = z - 1$, and apply (55) to (54), then we have the following inequality:

$$f_1(t,z) \le f(t,z) \le f_2(t,z), \tag{56}$$

where the two functions $f_1(t,z)$ and $f_2(t,z)$ are defined as follows:

$$f_1(t,z) = exp\Big[-t(z-1) + \frac{1}{2}\lambda^2\sigma_C^2(z-1)^2\Big], \tag{57}$$

and

$$f_2(t,z) = exp\Big[-t(z-1) + \frac{1}{2}(\sigma_l^2 + t)(z-1)^2\Big]. \tag{58}$$

Take the derivatives of (54), (57) and (58), we obtain

$$-\frac{\mu_l + t}{z^*} + \lambda\mu_C + \lambda^2\sigma_C^2(z^*-1) = 0, \tag{59}$$

$$-t + \lambda^2\sigma_C^2(z_1 - 1) = 0, \tag{60}$$

and

$$-t+\left(\sigma_l^2+t\right)\left(z_2-1\right)=0. \quad (61)$$

It follows from (56), the following inequalities should hold:

$$\inf_{z>1} f_1(t,z)=f_1(t,z_1)\le f_1(t,z^*)\le f(t,z^*), \quad (62)$$

$$\inf_{z>1} f(t,z)=f(t,z^*)\le f(t,z_2)\le f_2(t,z_2). \quad (63)$$

Combining (62) and (63), the following expression can be obtained from z_1 and z_2 given by (60) and (61) respectively,

$$f_1(t,z_1)=exp\left[-\frac{t^2}{2\lambda^2\sigma_C^2}\right]\le f(t,z^*)$$
$$\le exp\left[-\frac{t^2}{2\left(\sigma_l^2+t\right)}\right]=f_2(t,z_2). \quad (64)$$

Let t^*, t_1 and t_2 be the solutions that respectively satisfy the following three equations:

$$exp\left[-\frac{t_1^2}{2\lambda^2\sigma_C^2}\right]=f(t^*,z^*)=exp\left[-\frac{t_2^2}{2\left(\sigma_l^2+t_2\right)}\right]=\varepsilon.$$

Then, according to (64), we have

$$exp\left[-\frac{t^{*2}}{2\lambda^2\sigma_C^2}\right]\le f(t^*,z^*)=exp\left[-\frac{t_1^2}{2\lambda^2\sigma_C^2}\right]$$
$$=exp\left[-\frac{t_2^2}{2\left(\sigma_l^2+t_2\right)}\right]\le exp\left[-\frac{t^{*2}}{2\left(\sigma_l^2+t^*\right)}\right]. \quad (65)$$

Since those exponential functions in (65) are monotonically decreasing with t, we have

$$t_1\le t^*\le t_2. \quad (66)$$

Substituting $t=M-\mu_l$ into (66), we obtain

$$M_1\le M^*\le M_2. \quad (67)$$

Hence, the smallest integer M_1 that satisfies the following inequality is a lower bound of M^*:

$$exp\left[-\frac{\left(M_1-\mu_l\right)^2}{2\lambda^2\sigma_C^2}\right]\le\varepsilon,$$

and it can be explicitly expressed as follows:

$$M_1=\left\lceil\mu_l+\lambda\sigma_C\sqrt{2\log\varepsilon^{-1}}\right\rceil=\left\lceil\mu_l+\lambda\sigma_C\sqrt{2\alpha}\right\rceil. \quad (68)$$

Similarly, the smallest integer M_2 that satisfies the following inequality is an upper bound of M^*:

$$exp\left[-\frac{\left(M_2-\mu_l\right)^2}{2\left(\lambda^2\sigma_C^2+M_2\right)}\right]\le\varepsilon,$$

and it can be given as follows:

$$M_2=\left\lceil\mu_l+\log\varepsilon^{-1}+\sqrt{\left(\log\varepsilon^{-1}\right)^2+2\log\varepsilon^{-1}\left(\mu_l+\lambda^2\sigma_C^2\right)}\right\rceil$$
$$=\left\lceil\mu_l+\alpha+\sqrt{\alpha^2+2\alpha\sigma_l^2}\right\rceil. \quad (69)$$

We obtain (38) by combining (67)-(69).

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