# Stability and Delay Analysis of EPON Registration Protocol

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*Abstract***—The Ethernet passive optical network (EPON) has recently emerged as the mainstream of broadband access networks. The registration process of EPON, which is defined by the IEEE 802.3av standard, is a multi-point control protocol within the media access control layer. As with other contention-based channel access methods, such as ALOHA and CSMA, stability and delay are critical issues concerning the performances of implementing the protocol on systems with finite channel capacity. In this paper, the registration process of an EPON subscriber, called optical network units (ONUs), is modeled as a discrete-time Markov chain, from which we derive the fundamental throughput equation of EPON that characterizes the registration processes. The solutions of this characteristic equation depend on the maximum waiting time. The aim of our stability analysis is to pinpoint the region of the maximum waiting time that can guarantee a stable registration throughput and a bounded registration delay. For a maximum waiting time selected from the stable region, we obtain the expression of registration delay experienced by an ONU attempting to register. All analytic results presented in this paper were verified by simulations.**

*Index Terms***—EPON, registration process, IEEE 802.3av, stable region, registration delay.**

#### I. INTRODUCTION

**ESPITE** the growth of Internet traffic and the telecommunications backbone at an unprecedented pace, the access network between end-users and the core network remains the bottleneck for broadband integrated services. The passive optical network (PON) has recently emerged as one of the most promising candidates for broadband access networks, due to its simplicity, low cost, and capability of supporting various kinds of bandwidth-intensive services, such as video-ondemand (VoD), distance learning, and video conferencing [1].

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Fig. 1. Illustration of the EPON network.

Currently, two kinds of PONs, Ethernet PON (EPON) [2], [3] and Gigabit PON (GPON) [4], were issued by the IEEE and the ITU-T, respectively, and they have become the mainstream technology for fiber-to-the-home (FTTH) applications of the broadband access market [5]–[8].

The PON is a point-to-multipoint network. A tree-andbranch topology of a typical PON is illustrated in Fig. 1, which consists of an optical line terminal (OLT) at the central office, N optical network units (ONU) at the subscriber side, and an  $1: N$  optical coupler (OC) in between. The population of ONUs  $N$  is 32 or 64 in a 1G-EPON (IEEE 802.3ah [2]) or a 1G-GPON (ITU-T G.984 [4]), which can be extended to 256 or 512 in a 10G-EPON (IEEE 802.3av [3]) or a 10G-GPON (ITU-T G.987 [9]), respectively. In the next-generation PONs, e.g., Long-Reach PON (LR-PON), it is expected that the population size N will be further expanded to 1024 [10], [11] or 2048 [12], [13] to meet the ever-increasing demands of FTTH applications [14]–[16]. Based on an online scheduling [17]–[22] of the OLT, the ONUs share the bandwidth of the channel between the OLT and the OC in a time-divisionmultiplexing (TDM) manner. Thus, the OLT should collect information about all active ONUs in advance to establish this online scheduling through an ONU registration process, which is an essential element of all existing PON systems.

Similar to ALOHA [23] and CSMA [24], the ONU registration process is also a media access control (MAC) protocol within the family of IEEE 802.x stack. To commence a registration process, the OLT initiates a discovery window, in which all registered ONUs terminate their upstream transmissions, while unregistered ONUs randomly send the registration requests to the OLT if they attempt to be connected. As a result, the registration request of an ONU could be ruined if it collides with other randomly generated requests in the discovery window. The failed ONUs try again by sending registration requests in the next discovery window [1], [25]. The collisions among registration requests reduce the success probability or the throughput of the registration process [1], [26]. All existing PONs exploit the same registration protocol, except that different systems may employ different terminologies and use interaction messages with different lengths. The EPON registration processes employee the master/slave model of communication where one OLT unidirectionally controls over the registration processes of many ONUs. Due to the necessity of online scheduling of OLT, it is expected that this model of communication is likely to remain unchanged in the next generation LR-PON, which may be a pure TDM-PON with a tree-and-branch topology [18], [19] or a mixed time and wavelength division multiplexed PON (TWDM-PON) with a ring-and-spur topology [13], where multiple tree-and-branch sections at different wavelengths are connected to the main WDM ring via remote nodes. In the latter case, each section can be considered as an independent TDM-PON subsystem with respect to the registration process.

Parallel to other MAC protocols in the IEEE 802.x family, the registration protocol of EPONs may also encounter the throughput stability problem due to collisions among requests. Different from that of GPONs, the registration protocol of EPONs does not specify the maximum waiting time [27], which becomes the only adjustable parameter in the practical operation of the registration process. If the maximum waiting time is set too short, excessive collisions may lead to many reattempts accumulated by unsuccessful ONUs in the registration processes. Especially in the scenario of FTTH, a large number of ONUs are geographically clustered in a small residential area [28], as shown in Fig. 1, and each ONU may frequently turn on and off for the sake of power savings. In this case, the system may become unstable and a bounded registration delay cannot be guaranteed for each ONU. On the other hand, expanding the maximum waiting time of the registration process will inevitably reduce the bandwidth available for the upstream transmission of normal service. It is therefore essential to explore the stability and delay issues of the registration protocol. Although some previous works on the throughput analysis of the EPON registration protocol were reported in [25]–[27], [29], [30], and the registration delay was investigated in [30] based on simulations, a complete analysis to comprehend both stability and delay issues of the registration protocol remains open.

The research on the throughput and delay analysis of the MAC protocols, such as Aloha and CSMA, was pioneered by Abramsons work [23], and followed by the paper by Kleinrock and Tobagi [24]. Subsequently, many papers [31]–[35] demonstrated that the throughput of MAC protocols may become unstable if some relevant system parameters were not selected properly. Recently, the stability and delay issues of the MAC protocols were completely tackled in a sequence of papers [34], [36], in which the stability condition and bounded delay region of the system were determined by the analysis of a two-stage queuing model. Despite that, these queuing models cannot be directly applied to the analysis of the registration protocol of EPONs, because it has no external input traffic. The registration requests arriving at each discovery window include internally generated requests by the ONUs that were just turned on and those failed ONUs in the last discovery window.

Based on our previous throughput analysis presented in [26], we make the first attempt in this paper to model the behavior of ONUs, and analyze the stability and delay of the EPON registration protocol. In a discovery window, an ONU may be in one of the following states: online state, registration state, or offline state. Our analysis is based on the discretetime Markov chain that describes the state transitions of a tagged ONU. The aim of the analysis is twofold: determine the stable region of the maximum waiting time and estimate the bounded delay of the registration process in the stable region. Our results demonstrate that there is a stable region of the maximum waiting time in the steady state of the system. In this stable region, we show that the throughput and bounded delay of the registration protocol can be guaranteed. Furthermore, our results reveal the fact that it is unnecessary to arbitrarily enlarge the maximum waiting time as long as it is in the stable region because the improvement of registration delay is marginal, while the reduction of registration efficiency can be quite significant. We use 10G-EPON with 512 ONUs and LR-PON with 2048 ONUs as examples throughout this paper to illustrate our results, which demonstrate that our model is effective and practical in selecting a proper maximum waiting time for the registration process of different types of PONs.

The remainder of this paper is organized as follows. In Section II, we describe the registration process of EPON, and establish the discrete-time Markov chain to delineate the registration processes under the assumption of FTTH application. The main result is the derivation of the fundamental equation of EPON that characterizes the throughput of the system. In Section III, we study the stability of the registration protocol, and specify the stable region of the maximum waiting time for the practical implementation of EPON. In Section IV, we investigate the average registration delay experienced by an ONU attempting to register when the system is operating in the stable region. Section V provides a conclusion of this paper. For easy reference, the parameters employed in this paper are listed as follows.





Fig. 2. Registration processes of EPONs for the FTTH scenario.



 $\pi_R^s$ ,  $\pi_R^d$ , and  $\pi_R^u$ Solutions for the limiting probability  $\pi_R$ . d Registration delay of each ONU when the system is stable.

k Number of discovery windows elapsed until an attempting ONU could register successfully.  $\eta$  Registration efficiency.

Furthermore, we employ the following two examples throughout this paper to facilitate our discussions:

- Example 1: 10G-EPON where  $N=512$ ,  $Q=100 \mu s$ ,  $T=$ 500 ms and  $\tau_A = \tau_F = 600$  s;
- Example 2: LR-PON where  $N = 2048$ ,  $Q = 500 \mu s$ ,  $T = 500 \text{ ms}$ and  $\tau_A = \tau_F = 600$  s.

## II. MARKOV MODEL OF REGISTRATION PROCESSES

#### *A. Registration Processes*

The EPON registration processes, as defined in the IEEE 802.3av standard and illustrated in Fig. 2(a), alternate between a discovery window (marked by  $D$  in Fig. 2(a)) and a normal transmission window (marked by  $T$  in Fig. 2(a)) over time. The size of a transmission window is usually much larger than that of a discovery window. In each transmission window, the network provides broadband access services for the online ONUs based on a predetermined scheduling of the OLT. The normal access service is periodically interrupted by a discovery window, during which unregistered ONUs can make registration requests to the OLT if they attempt to access the network. The time interval between the start points of two consecutive discovery windows is referred to as *registration cycle*.

As plotted in Fig. 2(b), the OLT initiates a discovery window by broadcasting a discovery GATE message, which delivers the start time of the discovery slot and its length  $D$  to all ONUs [1]. After a one-trip propagation time, the GATE message arrives at the ONU. Let Q be the maximal one-trip propagation time from the OLT to the farthest ONU. In a 10G-EPON, the transmission distance is about 20 km, and thus the propagation delay  $Q$  is about 100  $\mu$ s [25]–[27]. In the future LR-PON, the transmission distance could be enlarged to 100 km [10]–[13], and the delay  $Q$  may increase to 500  $\mu$ s accordingly. When an ONU receives the GATE message, it may be in one of the following three states: online state A, offline state  $F$ , or registration state  $R$ . In the existing EPONs, an ONU has normal bandwidth access as long as it is in the online state A. In the next-generation PONs, however, an online ONU may go to sleep [18], [21] for the energy-saving purpose if it is temporarily idle. A sleeping ONU can wake up at any time and immediately transmit message [21] because it is still in online state A. That is, the ONU will ignore the GATE message if it is already in the online state A regardless it is sleeping or not. The ONU may be completely turned off and disconnected from the OLT if the user does not want to surf the Internet for a long period of time.

When a power-off ONU is turned on again, it will be ready for registration only after its components and devices were fully stabilized, which may take a certain amount of time [3], [18]. The holding time of the offline state  $F$  is therefore defined as the duration elapsed from the instant when the ONU is turned off to the moment when it is stabilized. After warming-up its components, the ONU will transit into the registration state  $R$ until it is successfully registered.

Upon receiving the GATE message, each ONU in the registration state sends back a request message REQ to the OLT after waiting for a random delay in the discovery slot [1]. The length of the REQ message is  $L < 3 \mu s$  in a 10G-EPON [1], [25]–[27]. According to the registration protocol, the random delay is uniformly chosen from the interval  $[0, \omega]$ , where  $\omega = D - L$ , and is called the *maximum waiting time* [27]. The REQ message arrives at the OLT after another one-trip propagation time. The discovery window should be large enough such that the OLT can receive all REQs. Therefore, the discovery window size is  $M = 2Q + D$ , the sum of the maximal round-trip propagation time and the discovery slot size.

A collision occurs if two or more REQs overlap in time when they arrive at the OLT. All REQs involved in the collision are ruined and the corresponding registrations are void. The ONU that fails to register remains invisible to the OLT, and it tries to register again in the next discovery window. According to the registration process defined in the IEEE 802.3av standard, a discrete-time Markov chain that characterizes the state transitions of a tagged ONU is described in the rest of this section.

#### *B. Discrete-Time Markov Chain*

The registration processes described above can be modeled as a discrete-time Markov chain. The application of the EPON system under consideration is FTTH, which is regarded as the mainstream of broadband access networks [1], [37]. In the scenario of FTTH, it is estimated that the number of ONUs in an EPON can reach as many as 512 in the near future [15], [16]. These ONUs are usually clustered in a small housing district [28], such that their distances to the OLT are almost the same. In addition, each ONU may frequently turn on and off to save power. Considering these points, the Markov chain is established under the following assumptions:

- 1) The ONUs are clustered in a small area, and the variation of the propagation delays from the ONUs to the OLT is ignored;
- 2) The behaviors of ONUs are statistically identical in steady state, and each unregistered ONU makes an independent registration request in a discovery window;
- 3) The online holding time of each registered ONU, denoted by  $t_A$ , is a negative exponential random variable with mean  $\tau_A = E[t_A];$
- 4) The offline holding time of each idle ONU, denoted by  $t_F$ , is also a negative exponential random variable with mean  $\tau_F = E[t_F];$
- 5) The registration cycle time  $T$  between the starting points of two consecutive discovery windows, as illustrated in Fig. 2(a), is assumed to be a constant.



Fig. 3. Discrete-time Markov chain of an ONU in EPON.

We define the state of a tagged ONU at the moment when it receives the GATE message from the OLT in each discovery window. The state transitions of this ONU can be described by the discrete-time Markov chain shown in Fig. 3. The three states  $A, F$ , and  $R$  of the Markov chain represent the registered online state, offline state, and registration state, respectively, of the ONU under consideration. From the state transition diagram, we know that the limiting probabilities  $\pi_A$ ,  $\pi_F$ , and  $\pi_R$  satisfy the following set of equations:

$$
\begin{cases}\n\pi_A(1 - P_{A,A}) = \pi_R P_{R,A} \\
\pi_F(1 - P_{F,F}) = \pi_A P_{A,F} + \pi_R P_{R,F} , \\
\pi_R(1 - P_{R,R}) = \pi_A P_{A,R} + \pi_F P_{F,R}\n\end{cases}
$$
\n(1)

where  $P_{x,y}$  is the transition probability for state  $x, y \in \{A, F, R\}$ .

The transition probabilities of the Markov chain can be derived from the above set of assumptions. Let  $t_n(n = 1, 2, \ldots)$ denote the starting point of the nth discovery window. Suppose that the tagged ONU is in state  $F$ , the offline state, at time  $t_n$ . It will participate in the registration process in the next discovery window if it is turned on before time  $t_{n+1}$ , or equivalently, the offline holding time  $t_F$  is less than the cycle time T. It follows from assumption 4 above that the transition probability from state  $F$  to state  $R$  is given by:

$$
P_{F,R} = Pr\{t_F < T\} = 1 - e^{-\frac{T}{\tau_F}}.\tag{2}
$$

Since the ONU cannot directly move from the offline state  $F$  at time  $t_n$  to the online state A at time  $t_{n+1}$  without registration, as plotted in Fig. 3, the probability that the ONU remains in state F at time  $t_{n+1}$  is given by:

$$
P_{F,F} = 1 - P_{F,R} = e^{-\frac{T}{\tau_F}}.
$$
 (3)

Suppose the ONU is in the online state A at time  $t_n$ . It could move to state R at time  $t_{n+1}$  if it is turned off and then turned on again before time  $t_{n+1}$ , which implies  $t_A + t_F < T$ . Thus, according to assumptions 3 and 4, we can calculate the transition probability  $P_{A,R}$  from state A to state R as follows:

$$
P_{A,R} = Pr\{t_A + t_F < T\}
$$
\n
$$
= Pr\{t_A < T\} Pr\{t_A + t_F < T|t_A < T\}
$$
\n
$$
= \left(1 - e^{-\frac{T}{\tau_A}}\right) p_{rer},\tag{4}
$$



Fig. 4. A successful registration message.

where  $p_{rer}$  is the probability that the ONU will be involved in the registration process in the next discovery window given that it is turned off before  $t_{n+1}$ . Again, from assumptions 3 and 4 above, we have:

$$
p_{rer} = Pr\{t_A + t_F < T | t_A < T\}
$$
\n
$$
= \frac{\tau_F \left(1 - e^{-\frac{T}{\tau_F}}\right) - \tau_A \left(1 - e^{-\frac{T}{\tau_A}}\right)}{(\tau_F - \tau_A) \left(1 - e^{-\frac{T}{\tau_A}}\right)}.
$$
\n
$$
(5)
$$

On the other hand, the ONU in online state A at time  $t_n$  could move to state F at time  $t_{n+1}$  if it is turned off but not turned on again before time  $t_{n+1}$ , which implies that  $t_A < T$  while  $t_A + t_F > T$ . Therefore, according to assumptions 3 and 4, the transition probability  $P_{A,F}$  from state A to state F is given as follows:

$$
P_{A,F} = Pr \{ (t_A < T), (t_A + t_F > T) \}
$$
  
=  $Pr \{ t_A < T \} \times Pr \{ t_A + t_F > T | t_A < T \}$   
=  $\left( 1 - e^{-\frac{T}{\tau_A}} \right) (1 - p_{rer}).$  (6)

From  $(4)$ – $(6)$ , we have:

$$
P_{A,A} = 1 - P_{A,R} - P_{A,F} = e^{-\frac{T}{\tau_A}}.
$$
 (7)

Suppose that the ONU is in the registration state R at time  $t_n$ . It could be in the online state A at time  $t_{n+1}$  if it is successfully registered in the nth discovery window and not turned off before time  $t_{n+1}$ . Therefore, the transition probability  $P_{R,A}$ from state  $R$  to state  $A$  is given as follows:

$$
P_{R,A} = Pr \{ (\text{Registration is successful}), (t_A \ge T) \}
$$

$$
= Pr \{ \text{Registration is successful} \} \times Pr \{ t_A \ge T \}
$$

$$
= p_{suc} \times e^{-\frac{T}{\tau_A}},
$$
(8)

where  $p_{suc}$  is the success probability of a registration request. In assumption 2 above, we assume that each unregistered ONU makes an independent registration request in a discovery window. For a sufficiently large population size  $N$ , the mean number of ONUs involved in each registration process is given by the following definition:

$$
G = N\pi_R. \tag{9}
$$

The following lemma provides the derivation of  $p_{succ}$ , which is similar to that of the pure Aloha system given in [23].

*Lemma 1:* The success probability of a registration request in a discovery window is given by:

$$
p_{suc} = e^{-\frac{2L}{\omega}G}.\tag{10}
$$

*Proof:* Suppose that a tagged REQ (registration request message) of length L starts at time  $t_0$  and ends at time  $t_1 =$  $t_0 + L$  in the discovery slot elapsed from time  $t = 0$  to time  $t =$  $\omega + L$ , as shown in Fig. 4. This registration request succeeds if no other request messages start in the vulnerable period. Considering the boundary of discovery slot, there are three kinds of vulnerable period described as follows:

- 1) If  $L \le t_0 \le \omega L$ , then the vulnerable period is from time  $t_0 - L$  to  $t_0 + L$ ;
- 2) If  $0 \le t_0 < L$ , when the REQ message overlaps into the head period, then the vulnerable period is from time 0 to  $t_0 + L;$
- 3) If  $\omega L < t_0 \leq \omega$ , when the REQ message overlaps into the tail period, then the vulnerable period is from time  $t_0 - L$ , to  $\omega$ .

Since the maximum waiting time  $\omega$  is much larger than the message length  $L$ , and the probability that a REQ message overlaps into head or tail period is negligible, we can ignore cases 2 and 3 above in the following derivation. Given that the starting point of a message is uniformly distributed in the time interval  $[0, \omega]$ , then the probability q that an arbitrary ONU will not interfere with this tagged registration message is given by:

$$
q = Pr\{\text{ONU not in state } R\} + Pr\{(\text{ONU in state } R),\}
$$

(request message not starting in vulnerable period)}

$$
\approx (1 - \pi_R) + \pi_R \left( 1 - \frac{2L}{\omega} \right)
$$

$$
= 1 - \frac{2L}{\omega} \pi_R.
$$
 (11)

According to assumption 2, we obtain:

$$
p_{suc} = q^{N-1} \cong \left(1 - \frac{2L}{\omega} \pi_R\right)^{N-1}
$$

$$
= \left[1 - \frac{1}{N-1} \left(\frac{2LN\pi_R}{\omega}\right) \frac{N-1}{N}\right]^{N-1}.
$$
 (12)

For  $N \gg 1$ , we have:

$$
p_{suc} \cong \left[1 - \frac{1}{N-1} \left(\frac{2L}{\omega}G\right)\right]^{N-1} \cong e^{-\frac{2L}{\omega}G}.\tag{13}
$$

-

The above derivation is consistent with the result that we obtained from the throughput analysis of EPON reported in [26]. It is possible that the ONU could also move into the offline state  $F$  if it is successfully registered in the *n*th discovery window but turned off before time  $t_{n+1}$ . Consequently, the transition probability  $P_{R,F}$  from state R to state F can be determined as follows:

$$
P_{R,F} = Pr \{ (\text{Registration is successful}), (t_A < T),
$$
\n
$$
(t_A + t_F > T) \}
$$
\n
$$
= p_{suc} \times Pr \{ (t_A < T), (t_A + t_F > T) \}
$$
\n
$$
= p_{suc} \times Pr \{ t_A < T \} \times Pr \{ t_A + t_F > T | t_A < T \}
$$
\n
$$
= p_{suc} \times \left( 1 - e^{-\frac{T}{\tau_A}} \right) \times (1 - p_{rer}). \tag{14}
$$

Again, from (8) and (14), we obtain the transition probability  $P_{R,R}$  that the ONU remains in state R as follows:

$$
P_{R,R} = 1 - P_{R,A} - P_{R,F}
$$
  
= 1 - p<sub>suc</sub> + p<sub>suc</sub> × (1 - e<sup>- $\frac{T}{\tau_A}$</sup> ) × p<sub>rer</sub>. (15)

In practice, the cycle time  $T$  is on the order of hundreds of milliseconds [1], [38], and it is much smaller than  $\tau_A$  and  $\tau_F$ , because users are not likely to turn on and off the ONUs very frequently. Based on this practical condition, we obtain the following approximations, which are helpful to make the computation of our model more tractable.

*Lemma 2:* If  $\tau_A$ ,  $\tau_F \gg T$ , then we have: A1)  $p_{rer} \approx 0$ ; A2)  $(\pi_A/\pi_F) \cong (\tau_A/\tau_F)$ .

*Proof:* Ignoring the higher order terms in the Taylor's series expansion  $e^{-x} \approx 1 - x + o(x)$  for  $|x| \ll 1$ , we obtain A1 from (5) as follows:

$$
p_{rer} \approx \frac{\tau_F \left[1 - \left(1 - \frac{T}{\tau_F}\right)\right] - \tau_A \left[1 - \left(1 - \frac{T}{\tau_A}\right)\right]}{\left(\tau_F - \tau_A\right)\left(1 - e^{-\frac{T}{\tau_A}}\right)} = 0. \tag{16}
$$

From the first and second balance equations of (1) and the transition probabilities given by (3), (6)–(8), and (14), we have:

$$
\frac{\pi_A}{\pi_F} = \frac{(1 - P_{F,F})P_{R,A}}{P_{A,F}P_{R,A} + (1 - P_{A,A})P_{R,F}} \\
= \frac{\left(1 - e^{-\frac{T}{\tau_F}}\right) \times e^{-\frac{T}{\tau_A}}}{\left(1 - e^{-\frac{T}{\tau_A}}\right) \times (1 - p_{rer})}.\n\tag{17}
$$

Similarly, we obtain A2 as follows:

$$
\frac{\pi_A}{\pi_F} \cong \frac{\left[1 - \left(1 - \frac{T}{\tau_F}\right)\right] \times \left(1 - \frac{T}{\tau_A}\right)}{\left[1 - \left(1 - \frac{T}{\tau_A}\right)\right]} = \frac{\tau_A \left(1 - \frac{T}{\tau_A}\right)}{\tau_F} \cong \frac{\tau_A}{\tau_F}.
$$
\n(18)

Given the transition probabilities  $(2)$ – $(8)$  and  $(14)$ ,  $(15)$  in the steady-state, the set of (1) can be simultaneously solved with the characteristic equation of throughput given in the following theorem.

*Theorem 1:* In the steady-state of the EPON registration process, the limiting probability  $\pi_R$  of the Markov chain satisfies the following characteristic equation:

$$
(1 - \pi_R)h = \pi_R e^{-\frac{2LN}{\omega}\pi_R},\tag{19}
$$

where  $h$  is a constant approximately given by:

$$
h \cong \frac{T}{\tau_A + \tau_F}.\tag{20}
$$

*Proof:* In the steady-state of the EPON registration process, from the first balance equation of (1), we obtain:

$$
\pi_A = \frac{\pi_R P_{R,A}}{1 - P_{A,A}}.\tag{21}
$$

Clearly, we have  $\pi_A + \pi_F + \pi_R = 1$ , because each ONU must be in one of the three states at the beginning of any discovery window. Substituting (21) into the second balance equation of (1), we obtain:

$$
\left(1 - \pi_R - \frac{\pi_R P_{R,A}}{1 - P_{A,A}}\right) (1 - P_{F,F}) = \frac{\pi_R P_{R,A}}{1 - P_{A,A}} P_{A,F} + \pi_R P_{R,F}.
$$
\n(22)

Now, we can derive (19) from (22) together with the transition probabilities given by (3), (6)–(8), and (14), where  $h$  is a constant given as follows:

$$
h = \frac{\left(1 - e^{-\frac{T}{\tau_A}}\right)\left(1 - e^{-\frac{T}{\tau_F}}\right)}{1 - e^{-\left(\frac{T}{\tau_A} + \frac{T}{\tau_F}\right)} - p_{rer}\left(1 - e^{-\frac{T}{\tau_A}}\right)}.\tag{23}
$$

The approximation of  $h$  given by (20) can be readily obtained from A1 of Lemma 2.

The above theorem gives the fundamental (19) of EPON that characterizes the throughput of the system. Multiplying both sides of (19) by  $N$ , we obtain:

$$
N(1 - \pi_R)h = N\pi_R e^{-\frac{2LN}{\omega}\pi_R}.
$$
 (24)

The right-hand side of (24) is equal to the average number of ONUs successfully registered per registration cycle,  $N \pi_R p_{succ}$ , or the *departure rate* of the registration process, denoted by  $\lambda_{out}$ . On the left-hand side,  $N(1 - \pi_R)$  is the average number of ONUs not in the registration state  $R$ . We want to demonstrate that the characteristic (19) is the balance equation between the *arrival rate*  $N(1 - \pi_R)h$  and the departure rate of registration requests, and the parameter h is the *attempt probability*, defined as the probability that an ONU not in the registration state will attempt to register in the next discovery window. This point can be elaborated by the integrated two-state Markov chain shown in Fig. 5.

In the steady-state, an ONU is either in the registration state R with probability  $\pi_R$ , or in the other two non-registration states, the online state  $A$  and the offline state  $F$ , with probability  $\pi_A + \pi_F = 1 - \pi_R$ . The discrete-time Markov chain shown in Fig. 3 is thus condensed into the two-state  $\{0,1\}$  Markov chain displayed in Fig. 5, where the registration state 1 corresponds to state  $R$ , while the non-registration state 0 represents the integration of state A and state F. Let random variable  $\chi_n$ 



Fig. 5. The integrated two-state Markov chain.

denote the state of an ONU at the time  $t_n$ . The transition probability from the non-registration state 0 to the registration state 1 is then defined by:

$$
P_{0,1} = Pr\{\chi_{n+1} = 1 | \chi_n = 0\}
$$
  
=  $Pr\{\chi_{n+1} = 1 | \text{state } A \text{ at } t_n \} Pr\{\text{state } A \text{ at } t_n | \chi_n = 0\}$   
+  $Pr\{\chi_{n+1} = 1 | \text{state } F \text{ at } t_n \}$   
 $\times Pr\{\text{state } F \text{ at } t_n | \chi_n = 0\}$   
=  $P_{A,R} \frac{\pi_A}{\pi_A + \pi_F} + P_{F,R} \frac{\pi_F}{\pi_A + \pi_F}.$  (25)

Also, from (25) and the definition of limiting probability  $\pi_A$  +  $\pi_F = 1 - \pi_R$ , we have:

$$
(1 - \pi_R)P_{0,1} = \pi_A P_{A,R} + \pi_F P_{F,R}.
$$
 (26)

From the third balance equation of (1) and the transition probability given by (15), we have:

$$
(1 - \pi_R)P_{0,1} = \pi_A P_{A,R} + \pi_F P_{F,R}
$$
  
=  $\pi_R (1 - P_{R,R})$   
=  $\pi_R p_{suc} \left[ 1 - \left( 1 - e^{-\frac{T}{\tau_A}} \right) \times p_{rer} \right]$   
 $\approx \pi_R p_{suc}.$  (27)

The last step is due to the approximation A1 given in Lemma 2. Substituting (2) and (4) into (25), from Lemma 2, we obtain:

$$
P_{0,1} = P_{A,R} \frac{\pi_A}{\pi_A + \pi_F} + P_{F,R} \frac{\pi_F}{\pi_A + \pi_F}
$$
  
\n
$$
\approx \left(1 - e^{-\frac{T}{\tau_A}}\right) \frac{p_{rer} \tau_A}{\tau_A + \tau_F} + \left(1 - e^{-\frac{T}{\tau_F}}\right) \frac{\tau_F}{\tau_A + \tau_F}
$$
  
\n
$$
\approx \frac{T}{\tau_A + \tau_F}.
$$
\n(28)

Comparing (19) and (27), the above expression (28) confirms the fact that the attempt probability  $h$  given by (20) is indeed the transition probability  $P_{0,1}$  that an ONU not in the registration state will attempt to register in the next discovery window, and  $N(1 - \pi_R)h$  is the traffic that arrives per registration cycle.

The accuracy of our analytic result is confirmed by simulation, of which the method is presented at the end of Section IV. For the Example 1 presented in Section I, the exact value of the attempt probability h given by (23) is  $4.1675 \times 10^{-4}$  and the approximation given by (20) is  $4.1667 \times 10^{-4}$ , while the result obtained by simulation is  $4.1666 \times 10^{-4}$ .

#### III. STABILITY CONDITIONS

In this section, we explore the stability analysis of the EPON registration protocol. For a given set of EPON parameters, Theorem 1 states that the throughput of the protocol is determined by the maximum waiting time  $\omega$  by solving (19) for the limiting probability  $\pi_R$ . Intuitively,  $\omega$  has to be sufficiently large to alleviate contentions among registration requests generated by ONUs.

Consider the EPON of Example 1, the limiting probability  $\pi_R$  calculated from (19) can be as small as  $4.18 \times 10^{-4}$  and the steady-state registration throughput is  $\lambda_{out} \approx 0.2133$  when  $ω = 250 \mu$ s, while the limiting probability  $π<sub>R</sub> ≅ 1$  and  $λ<sub>out</sub> ≅$ 0 when  $\omega = 2 \mu s$ . This example indicates that the ONU has almost no chance to succeed in the registration process if the maximum waiting time  $\omega$  is too small. When the limiting probability  $\pi_R$  is very close to 1, nearly all the ONUs participate in the registration process, and eventually the EPON collapses. The aim of our stability analysis is to specify the stable region of the maximum waiting time  $\omega$ , in which the limiting probability  $\pi_R$  can be sufficiently small and the EPON system has a stable throughput.

We show in Appendix B that the solutions of the (19) for the limiting probability  $\pi_R$  depend on the regions of  $\omega$  specified by the following two critical parameters:

$$
\omega_0 = -\frac{2LNW_0(\alpha)}{\left[1 - W_0(\alpha)\right]^2},\tag{29}
$$

$$
\omega_{-1} = -\frac{2LNW_{-1}(\alpha)}{\left[1 - W_{-1}(\alpha)\right]^2},\tag{30}
$$

where  $\alpha = -eh$  is a constant, and  $W_0(\alpha)$  in (29) and  $W_{-1}(\alpha)$ in  $(30)$  are the two possible values of the Lambert W function, which is briefly described in Appendix A. These values of the Lambert W function are defined in the range of  $\alpha \geq -e^{-1}$ or equivalently  $h \le e^{-2}$ . According to (20), this condition imposes the following implementation requirement:

$$
T \le \frac{\tau_A + \tau_F}{e^2},\tag{31}
$$

which should normally hold in most practical EPON systems because typically we have  $\tau_A \gg T$  and  $\tau_F \gg T$ . Later in this section, we show that a strictly stable registration protocol actually requires a stricter condition on the attempt probability h.

The limiting probability  $\pi_R$  and the registration throughput  $\lambda_{out}$  with 90% confidential interval versus the maximum waiting time  $\omega$  of the discovery window is plotted in Fig. 6, in which we use Example 1 for Fig. 6(a) and (b) and Example 2 for Fig. 6(c) and (d). The number of solutions of (19) changes with respect to  $\omega$ . In Appendix B, we show the characteristic (19) has one solution if  $\omega < \omega_0$  or  $\omega > \omega_{-1}$ , and multiple solutions if  $\omega_0 \leq \omega \leq \omega_{-1}$ . Using parameters of Example 1, we elaborate the properties of these solution regions in the following.

#### *A. Saturated Region*

In the region  $\omega < \omega_0$ , the (19) of the limiting probability  $\pi_R$  has a unique solution, denoted as  $\pi_R^s$ . In this region, the maximum waiting time  $\omega$  is so small that almost all registration



Fig. 6. Different regions of  $\omega$  for limiting probability and steady-state throughput. (a)  $N = 512$ ; (b)  $N = 512$ ; (c)  $N = 2048$ ; (d)  $N = 2048$ .

requests will be ruined due to contention. Consequently, the solution  $\pi_R^s$  is very close to 1, and the throughput  $\lambda_{out} \cong 0$ , as Fig. 6 displays. Consider the extreme case when  $\omega = 0$ . It is easy to show from (19) that the limiting probability  $\pi_R^s$ approaches 1 and  $\lambda_{out}$  vanishes. This phenomenon is also revealed by the simulation results displayed in Fig. 7(a), where the two curves, respectively, correspond to 0% and 60% initial ONUs in state R with  $N = 512$  and  $\omega = 2 \mu$ s. In both cases, the limiting probability  $\pi_R$  converges to 1. Eventually, the system will be on the verge of collapse after a finite number of registration cycles.

# *B. Unpredictable Region*

In the region  $\omega_0 < \omega < \omega_{-1}$ , the (19) of the limiting probability  $\pi_R$  has three solutions, denoted as  $\pi_R^s$ ,  $\pi_R^d$  and  $\pi_R^u$ , where  $\pi_R^s > \pi_R^u > \pi_R^d$ , as shown in Fig. 6(a). It is a well-known property of an ergodic Markov chain that the limiting probability must be unique [39], [40], thus the performance of ONUs is unpredictable in this region because the system is obviously not ergodic. For the Example 1 with  $\omega = 150 \mu s$ , we first obtain the solution  $\pi_R^u = 43.61\%$  by solving (19). However, the simulation results indicate that the limiting probability  $\pi_R$  can either converge to  $\pi^s_R = 99.89\%$  or  $\pi^d_R = 0.042\%$ , as displayed in Fig. 7(b), with the same  $43.61\%$  ONUs in state R initially. Therefore, the registration protocol may result in different throughputs depending on the steady-state limiting probability  $\pi_R$ , as demonstrated in Fig. 6(b). Such bistable behavior is quite similar to that presented in the classical ALOHA-type networks [31]. For this reason, the performance of any individual ONU cannot be guaranteed in this unpredictable region.

## *C. Stable Region*

In the region  $\omega > \omega_{-1}$ , we show in Appendix B that the (19), which specifies the limiting probability of  $\pi_R$ , has only



Fig. 7. Simulation results by using parameters of Example 1 with different maximum waiting time  $\omega$ . (a) Saturated region:  $\omega < \omega_0$ . (b) Unpredictable region:  $\omega_0 < \omega < \omega_{-1}$ . (c) Stable region:  $\omega > \omega_{-1}$ .

one solution that converges to  $\pi_R^d$ . Accordingly, the system can achieve a stable throughput  $\lambda_{out}$ , as shown in Fig. 6(b). Presumably, a stable registration protocol requires a very small limiting probability  $\pi_R$ . We show in the following that the attempt probability h must also be kept small even if the maximum waiting time selected from the stable region  $\omega$  $\omega_{-1}$  is large enough. Intuitively, if users behave maliciously and turn on and off ONUs too frequently, meaning that  $\tau_A + \tau_F$  is too small, then the stable throughput may be achieved at the expense of a large mean delay.

If the system is stable, and the limiting probability  $\pi_R$ converges to  $\pi_R^d \ll 1$ , then we should be able to approximately rewrite (19) as follows:

$$
(1 - \pi_R)h \cong \pi_R \left(1 - \frac{2LN}{\omega} \pi_R\right),\tag{32}
$$

from which we obtain the following approximation of the limiting probability  $\pi_R$ :

$$
\pi_R \cong \frac{1 - \sqrt{1 - \frac{8LNh}{(1+h)^2 \omega}}}{4LN} (1+h)\omega.
$$
 (33)

The maximum waiting time  $\omega$  should satisfy the following condition if the above expression (33) is valid:

$$
\omega \ge \frac{8LNh}{(1+h)^2}.\tag{34}
$$

Thus, the stable condition of an EPON registration protocol is formally defined as follows:

*Definition 1:* The EPON registration protocol is stable if the maximum waiting time  $\omega$  is selected from the range  $\omega$  $\omega_{-1}$ . Moreover, the protocol is strictly stable if the attempt probability  $h$  satisfies the following inequality:

$$
\omega > \omega_{-1} > \frac{8LNh}{(1+h)^2}.\tag{35}
$$

In the following corollary, we show the above inequality (35) induces a necessary condition for a strictly stable registration protocol.

*Corollary 1:* A strictly stable registration protocol satisfies the following necessary condition:

$$
h < \frac{1}{16}.\tag{36}
$$

*Proof:* According to (30), the stable condition requires that:

$$
\omega \ge \omega_{-1} = -\frac{2LNW_{-1}(\alpha)}{\left[1 - W_{-1}(\alpha)\right]^2}.\tag{37}
$$

Thus, the following condition on the attempt probability  $h > 0$  ensures that both inequalities (35) and (37) can always be satisfied:

$$
-\frac{2LNW_{-1}(\alpha)}{\left[1 - W_{-1}(\alpha)\right]^2} > 8LNh > \frac{8LNh}{(1+h)^2},\tag{38}
$$

which is equivalent to:

$$
h < -\frac{W_{-1}(\alpha)}{4\left[1 - W_{-1}(\alpha)\right]^2}.\tag{39}
$$

We know from Fig. 11 in Appendix A that  $W_{-1}(\alpha) = -c^2$  for some  $c \geq 1$ , therefore the above inequality implies that:

$$
h < \frac{c^2}{4(1+c^2)^2} = \frac{1}{4\left(\frac{1}{c} + c\right)^2} \le \frac{1}{16}.\tag{40}
$$

-

Using the same EPON parameters of Example 1, we show in Fig. 8(a) that parameter  $\omega_{-1}$  increases with respect to h. In particular, parameter  $\omega_{-1}$  skyrockets when  $h > 1/16$ , which clearly demonstrates the necessity of the condition given by (36) for a strictly stable registration protocol.

We know from  $(19)$  that the arrival rate of registration requests per cycle time T is  $N(1 - \pi_R)h$ . For a given registration cycle T, a smaller attempt probability h implies larger  $\tau_A$ and  $\tau_F$ , meaning that the users tend to switch the ONUs on and off less frequently. Therefore, the ONUs are less likely to engage in a registration process. As a result, the arrival traffic for registration requests becomes smaller, which in turn decreases the probability of collision. Therefore, for a smaller attempt probability h, a smaller maximum waiting time  $\omega$  is required to moderate the collisions. This point is reinforced in the following corollary.

700

600

500

400  $(\mu s)$ 

200

100

 $\mathbf{0}$ 

 $10^{\circ}$ 

 $\omega$ <sub>-1</sub> 300

 $\pi$  when  $\omega = \omega$ .

 $10^{-2}$ 

 $\pi_R$ 

 $0.2$ 

 $0.1$ 

 $0.0$ 

 $10<sup>3</sup>$ 

 $10<sup>6</sup>$ 

 $(a)$ Fig. 8. (a) Parameter  $\omega_{-1}$  and (b) the limiting probability  $\pi_R$  versus attempt probability h.

 $\boldsymbol{h}$ 

 $\overline{16}$ 

 $10^{-}$ 

 $h' = e^{-2}$ 

*Corollary 2:* Suppose the maximum waiting time is selected from the region  $\omega > \omega_{-1}$ . Then the unique limiting probability  $\pi_R$  is bounded by:

 $10^{-2}$ 

$$
\frac{h}{1+h} \le \pi_R \le -\frac{W_0(\alpha)}{1 - W_0(\alpha)} \cong eh,\tag{41}
$$

where the approximation of the upper bound is valid for small attempt probability h.

*Proof:* In Appendix B, we prove that if  $\omega > \omega_{-1}$  then the (19), which specifies the limiting probability of  $\pi_R$ , has a unique solution in the interval  $[0, -(W_0(\alpha)/1 - W_0(\alpha))]$ . In Appendix A, we show that  $W_0(\alpha)$  has the following series expansion:

$$
W_0(\alpha) = \sum_{i=1}^{\infty} \frac{(-i)^{i-1}}{i!} \alpha^i = \alpha - \alpha^2 + \frac{3}{2} \alpha^3 - \frac{8}{3} \alpha^4 + \cdots,
$$
\n(42)

which is approximately equal to  $\alpha = -eh$  when the attempt probability  $h$  is small. That is, if the maximum waiting time  $\omega$  is large enough and the attempt probability h is small, i.e.,  $-\alpha \ll 1$ , then we have:

$$
\pi_R \ll -\frac{W_0(\alpha)}{1 - W_0(\alpha)} \approx -W_0(\alpha) \approx -\alpha = eh.
$$
 (43)

When the maximum waiting time  $\omega$  approaches infinity, we obtain the lower bound of the limiting probability  $\pi_R$  from the characteristic (19) as follows:

$$
\pi_R \to \frac{h}{1+h} \quad \text{as} \quad \omega \to \infty. \tag{44}
$$

-The above corollary implies that the limiting probability  $\pi_R$ cannot be further reduced by increasing a sufficiently large maximum waiting time  $\omega$ . The simulation result displayed in Fig. 7(c) confirms the fact that the limiting probability  $\pi_R$ converges to the following lower bound given by (41):

$$
\frac{h}{1+h} = \frac{\frac{0.5}{1200}}{1 + \frac{0.5}{1200}} \approx 4.165 \times 10^{-4},\tag{45}
$$

when the maximum waiting time is set to equal  $\omega = 240 \mu s$ , which is only slightly larger than the minimal requirement  $\omega_{-1} = 233.4 \ \mu s$  of the stable region when  $N = 512$ , regardless of the initial fraction of ONUs in state R.

 $(b)$ 

 $h^* =$  $16$  $\boldsymbol{h}$ 

 $10^{-1}$  $h' = e^{-2}$ 

The limiting probability  $\pi_R$  versus attempt probability h is plotted in the Fig. 8(b), which shows that  $-\alpha = eh$  almost coincides with the upper bound  $-(W_0(\alpha)/1 - W_0(\alpha))$  when the attempt probability  $h$  is small, especially when it is less than 1/16. Note that the limiting probability  $\pi_R$  displayed in Fig. 8(b) exhibits similar behavior of  $\omega_{-1}$  depicted in Fig. 8(a). In the region  $h < 1/16$ , the system is strictly stable and the limiting probability  $\pi_R$  approaches the lower bound. When  $h > 1/16$ , by contrast, the limiting probability  $\pi_R$  quickly approaches the upper bound.

#### IV. REGISTRATION DELAY

In this section, we analyze the registration delay experienced by an ONU attempting to register. The registration delay, denoted by d, is defined as the duration from the time an ONU is turned on until it is successfully registered. The components of a typical registration delay are illustrated in Fig. 9, in which the symbols involved are defined as follows:

- $t_{on}$  Time when an ONU is switched on.
- $t_{next}$  Time when the ONU receives the GATE in the next discovery window after it is switched on.
- $d_r$  Residual waiting delay, the time between  $t_{on}$  and  $t_{next}$
- $d_a$  Attempting delay, the duration of the registration cycles elapsed before the ONU is successfully registered.
- $t_{last}$  Time when the GATE arrives at the ONUs in the last discovery window before the tagged ONU is switched on.
- Z Interval between  $t_{last}$  and  $t_{on}$ ,  $Z = T d_r$ .

An ONU switched on at time  $t_{on}$  can only make its registration attempt at the time when it receives the GATE in the next discovery window, denoted by  $t_{next}$ . The delay between

Lower Bound:

 $\boldsymbol{h}$ 

 $1+h$ 

 $10^{\circ}$ 



Fig. 9. Components of the registration delay.

 $t_{on}$  and  $t_{next}$  is called *residual waiting delay*, denoted by  $d_r$ . If the attempt fails, the ONU tries again in the next registration cycle. We define the *attempting delay*, denoted by  $d_a$ , as the duration of the registration cycles elapsed before the successful registration is achieved. As shown in Fig. 9, the registration delay is the sum of these two components:  $d = d_r + d_a$ , and the average registration delay is given by:

$$
E[d] = E[d_r] + E[d_a].\tag{46}
$$

We first estimate the mean residual waiting time in the following lemma.

*Lemma 3:* The approximate mean residual waiting time is given by:

$$
E[d_r] \cong \frac{T}{2}.\tag{47}
$$

*Proof:* Recall that T is the registration cycle time. As illustrated in Fig. 9, we define the time interval  $Z = T - d_r$ as the interval between the moment when the ONU receives the GATE message in the last discovery window, denoted by  $t_{last}$ , and the time  $t_{on}$  that the ONU is turned on. Also, the unregistered ONU may be in state F or in state A at  $t_{last}$ , and it shifts to state R before or at time  $t_{next}$ . Thus, the distribution of Z can be derived as follows:

$$
P\{Z < z\} = P\{(Z < z), (F \to R)\}\
$$
\n
$$
+ P\{(Z < z), (A \to R)\}\
$$
\n
$$
= P\{Z < z|F \to R\} P\{F \to R\}\
$$
\n
$$
+ P\{Z < z|A \to R\} P\{A \to R\}, \quad (48)
$$

where the event  $i \to R$ , for  $i \in \{F, A\}$ , indicates that the ONU is in state i at time  $t_{last}$  and it engages in the registration process at time  $t_{next}$ . In the steady-state of the system, we have:

$$
P\{F \to R\} = \frac{\pi_F P_{F,R}}{\pi_F P_{F,R} + \pi_A P_{A,R}},\tag{49}
$$

$$
P\{A \to R\} = \frac{\pi_A P_{A,R}}{\pi_F P_{F,R} + \pi_A P_{A,R}}.\tag{50}
$$

Since  $\tau_A \gg T$  and  $\tau_F \gg T$ , from (2), (4), and the approximation A1 given in Lemma 2, we have:

$$
\frac{P_{A,R}}{P_{F,R}} = \frac{\left(1 - e^{-\frac{T}{\tau_A}}\right)p_{rer}}{1 - e^{-\frac{T}{\tau_F}}} \approx 0.
$$
 (51)

That is, the transition probability  $P_{A,R}$  is negligible when it is compared to  $P_{F,R}$ , which implies  $P\{F \to R\} \approx 1$  and  $P{A \rightarrow R} \approx 0$ . We then immediately obtain the following approximation from (48):

$$
P\{Z < z\} \cong P\{Z < z|F \to R\}.\tag{52}
$$

Given that the transition is from state  $F$  to state  $R$ , the duration  $z$  is the conditional offline holding time of the ONU. Thus, the conditional probability (52) can be expressed as follows:

$$
P\{Z < z \mid F \to R\} = \frac{\int_0^z \frac{1}{\tau_F} e^{-\frac{t}{\tau_F} dt}}{\int_0^T \frac{1}{\tau_F} e^{-\frac{t}{\tau_F}} dt} = \frac{1 - e^{-\frac{z}{\tau_F}}}{1 - e^{-\frac{T}{\tau_F}}} \cong \frac{z}{T}.\tag{53}
$$

The last approximation is because  $\tau_F \gg T > z$ . Furthermore, we know from  $(53)$  that the random variable Z is approximately uniformly distributed in the interval  $[0, T]$  with mean  $E[Z] \cong T/2$ . As a result, the average residual waiting time is given by:

$$
E[d_r] = E[T - Z] \cong \frac{T}{2}.
$$
\n(54)

-

Let  $k(k = 1, 2, 3, ...)$  be the number of discovery windows elapsed until an attempting ONU could register successfully. When  $k = 1$ , the attempting delay of the ONU is the duration of a single discovery window minus the transmission delay Q. When  $k = 2$ , the attempting delay of the ONU consists of a registration cycle and a single discovery window minus Q. In general, as illustrated in Fig. 9, the ONU spends  $k - 1$ registration cycles and a single discovery window minus Q on the registration. Hence, the attempting delay is given by:

$$
d_a = M - Q + (k - 1)T,
$$
\n(55)

where  $M$  is the discovery window size defined in Section II-A. Summarizing the above discussions, the mean registration delay is provided in the following theorem.

*Theorem 2:* Suppose the maximum waiting time is selected in the stable region  $\omega > \omega_{-1}$  or the saturated region  $\omega > \omega_0$ . The average registration delay that an ONU experiences in the registration processes is given by:

$$
E[d] \cong \left[\frac{\pi_R}{(1 - \pi_R)h} - \frac{1}{2}\right]T.
$$
 (56)

If the registration protocol is stable, the average delay is upper bounded by

$$
E[d] \le 6.89T. \tag{57}
$$

*Proof:* We know from (55) that the average attempting delay is given by:

$$
E[d_a] = M - Q + (E[k] - 1)T \cong (E[k] - 1)T, \quad (58)
$$



Fig. 10.  $E[d]$  and  $\eta$  with 90% confidential interval versus  $\omega$  when  $\omega \ge \omega_{-1}$ . (a)  $N = 512$ ; (b)  $N = 2048$ .

where the discovery window size  $M - Q$  can be ignored because it is typically much smaller than the cycle time T. Recall that, in the steady-state of EPON, an ONU attempting to register succeeds in a discovery window with probability  $p_{suc}$ . Thus, k is a geometric random variable with parameter  $p_{suc}$ . From (19), we have:

$$
E[k] = \frac{1}{p_{suc}} = \frac{\pi_R}{(1 - \pi_R)h}.
$$
 (59)

It follows from (58) and (59) that:

$$
E[d_a] \cong \left[\frac{\pi_R}{(1-\pi_R)h} - 1\right]T.
$$
 (60)

Substituting (47) and (60) into (46), we immediately obtain (56).

If the system is stable, we know from Corollary 2 that the limiting probability  $\pi_R$  is upper bounded by  $-(W_0(\alpha)/1 W_0(\alpha)$ ). Moreover, we know from (56) that the mean delay is upper bounded by:

$$
E[d] \cong \left[\frac{\pi_R}{(1-\pi_R)h} - \frac{1}{2}\right]T \le \left\{\frac{-\frac{W_0(\alpha)}{1-W_0(\alpha)}}{\left[1 + \frac{W_0(\alpha)}{1-W_0(\alpha)}\right]h} - \frac{1}{2}\right\}T
$$

$$
= \left[-\frac{W_0(\alpha)}{h} - \frac{1}{2}\right]T = \left[\frac{eW_0(\alpha)}{-eh} - \frac{1}{2}\right]T
$$

$$
= \left[\frac{eW_0(\alpha)}{\alpha} - \frac{1}{2}\right]T.
$$
(61)

According to the definition of the Lambert W function  $\alpha =$  $W_0(\alpha)e^{W_0(\alpha)}$  given in Appendix A, we have:

$$
E[d] \le \left[\frac{eW_0(\alpha)}{\alpha} - \frac{1}{2}\right]T = \left[\frac{eW_0(\alpha)}{W_0(\alpha)e^{W_0(\alpha)}} - \frac{1}{2}\right]T
$$

$$
= \left[e^{1-W_0(\alpha)} - \frac{1}{2}\right]T \le \left(e^2 - \frac{1}{2}\right) \cong 6.89T. \quad (62)
$$

-

From Corollary 2, we know that the limiting probability  $\pi_R$ is close to the lower bound if the system is strictly stable. Thus, for a large  $\omega > \omega_{-1}$  and a sufficiently small h in the strictly stable region, substituting (44) into (56), we have the following ideal mean delay:

$$
E[d] \cong \left[\frac{\pi_R}{(1-\pi_R)h} - \frac{1}{2}\right] T \cong \left[\frac{\frac{h}{1+h}}{\left(1 - \frac{h}{1+h}\right)h} - \frac{1}{2}\right] T = \frac{T}{2}.
$$
\n(63)

On the other hand, the registration delay is very large if the maximum waiting time  $\omega$  is in the saturated region  $\omega$  <  $ω_0$ . When  $ω$  is very small and the limiting probability  $π_R$ approaches 1, the average registration delay  $E[d]$  given by (56) may not be bounded. This property of an unstable registration protocol has been confirmed by our simulation results, which demonstrated the fact that an ONU cannot successfully register even after tens of thousands of registration cycles when  $\omega < \omega_0$ .

Intuitively, the average registration delay  $E[d]$  decreases with the increase of the maximum waiting time  $\omega$  in the stable region  $\omega > \omega_{-1}$ . This property is confirmed by the simulation results and the theoretical results plotted in Fig. 10, where we use the parameters in Example 1 for Fig. 10(a) and those in the Example 2 for Fig. 10(b). Also, we plot the simulation results of  $E[d]$  and  $\eta$  with 90% confidential interval. The predominate delay component is  $E[k]$ , the average number of discovery windows elapsed until the successful registration is achieved. With the increasing of  $\omega$ , the probability  $p_{suc}$  that a registration request is successful in a discovery window also increases, and thus  $E[k]$  decreases. However, Fig. 10 also shows that the reduction of the delay by increasing  $\omega$  is marginal. For instance, in Fig. 10(a), when  $\omega$  increases from 233.4  $\mu$ s to 700  $\mu$ s, more than doubling, the average delay  $E[d]$  only decreases by 0.67%, from 252.5 ms to 250.8 ms. The same thing happens in Fig. 10(b). This point can be explained as follows. In the strictly stable region, the limiting probability  $\pi_R$  is already very small and, thus, can only be reduced slightly by increasing  $\omega$ , as demonstrated by (44). It follows from (59) that the improvement in  $p_{suc}$ , or equivalently the reduction of  $E[k]$ , is insignificant. Consequently, the reduction of  $E[d]$  by increasing  $\omega$  is negligible, as predicted by (63).

In contrast, the increase of  $\omega$  not only shrinks the bandwidth available for normal transmission, but also affects the registration efficiency. In [1], [26], this indicator, denoted by  $\eta$ , is defined as the ratio of the number of successful registrations to the discovery window size and is given as follows:

$$
\eta = \frac{N\pi_R p_{suc}}{M} = \frac{Nh(1 - \pi_R)}{2Q + D} \cong \frac{Nh}{2Q + \omega + L},\qquad(64)
$$

where the second equality is obtained from (19) and the last approximation is due to the condition  $\pi_R \ll 1$  in the strictly stable region. A comparison of  $E[d]$  in (63) and  $\eta$  in (64) shows that the efficiency  $\eta$  drops much faster than then mean delay  $E[d]$  in respect to the increase of  $\omega$  in the strictly stable region. This point is also confirmed by the simulation results displayed in Fig. 10, which shows that the efficiency  $\eta$  is reduced sharply with the increase of  $\omega$ . Thus, a marginal reduction of the delay is achieved at the expense of a substantial degradation of registration efficiency  $\eta$ .

The analytic results presented in this paper are all verified by simulations. Our simulations are based on the event-driven approach described in [41], where the events, such as ONU power-off and message sending, are scheduled according to their time stamps. At the beginning of each simulation, we randomly generate the ONUs in online state A, offline state  $F$ , and registration state  $R$ . When an ONU successes in a discovery window, we calculate an online holding time  $t_A$  for this ONU according to the negative exponential distribution with mean  $\tau_A$ . Similarly, we determine an off-line holding time  $t_F$  according to the negative exponential distribution with mean  $\tau_F$  for the ONU when it is turned off. During every discovery window, upon receiving the GATE message from the OLT, each ONU sends their REQ message after a random delay, which is a uniformly distributed random variable in the interval  $[0, \omega]$ .

#### V. CONCLUSION

In this paper, we analyze the stability and delay of the EPON registration protocol. We first establish a model of the subscribers by using a discrete-time Markov chain, and then derive the characteristic function to delineate the throughput of the registration process. Solving the characteristic function, we obtain the region of the maximum waiting time to make the registration protocol stable. If the maximum waiting time is selected from this region, we show that a stable registration throughput and a bounded registration delay can be guaranteed. Our results also indicate that it is unnecessary to arbitrarily enlarge the maximum waiting time as long as it is in the stable region since the improvement of registration delay is marginal, yet the reduction of registration efficiency is quite significant.

In the analysis presented in this paper, we consider the typical urban application of EPON where the ONUs are clustered in a



Fig. 11. The Lambert W function.

small area, such that the difference of propagation delays from the OLT to ONUs can be ignored for the sake of mathematical simplicity. Despite that, our model still can capture the main characteristics of EPON registration processes, from which we obtain the stability conditions of the EPON protocol. In some prospective applications in rural areas, however, the dwelling units may be sparely distributed over the vicinity of an EPON system. If the propagation delay becomes a critical issue in such scenarios, it is feasible to include variable delays in our Markov model, which could be an interesting research topic in the future.

#### APPENDIX A LAMBERT W FUNCTION

The Lambert W function  $W(z)$ , which was first considered by J. Lambert [42], is defined as the function satisfying:

$$
W(z)e^{W(z)} = z.\t\t(65)
$$

The Lambert W function  $W(z)$  is plotted in Fig. 11. If z is real and  $-e^{-1} < z < 0$ ,  $W(z)$  has two real values: the principal branch  $W_0(z) \in [-1,0)$ , and the other branch  $W_{-1}(z) \in$  $(-\infty, -1]$ . If z is real and  $z > 0$ ,  $W(z)$  has only a branch  $W_0(z) \in [0,\infty).$ 

Reference [42] shows that, according to the Lagrange inversion theorem,  $W_0(z)$  has the following series expansion:

$$
W_0(z) = \sum_{i=1}^{\infty} \frac{(-i)^{i-1}}{i!} z^i = z - z^2 + \frac{3}{2} z^3 - \frac{8}{3} z^4 + \cdots,
$$
\n(66)

and  $W_{-1}(z)$  has the following series expansion:

$$
W_{-1}(z) = \sum_{i=0}^{\infty} \mu_i y^i = -1 + y - \frac{1}{3}y^2 + \frac{11}{72}y^3 - \frac{43}{540}y^4 + \cdots,
$$
\n(67)

where  $y = -\sqrt{(2(ez + 1))}$ . The coefficient  $\mu_i$  is given in [42].

# APPENDIX B SOLUTIONS OF THE CHARACTERISTIC EQUATION

In this appendix, we explore the solutions of the following characteristic equation that Theorem 1 establishes:

$$
(1 - \pi_R)h = \pi_R e^{-\frac{2LN}{\omega}\pi_R},\tag{68}
$$

Define the auxiliary function:

$$
F(x,\omega) = (1-x)h - xe^{-\frac{2LN}{\omega}x},
$$
\n(69)

Furthermore, this auxiliary function is denoted as  $F(x, \omega) =$  $f_{\omega}(x)$  for a fixed  $\omega$ , and  $F(x,\omega) = g_x(\omega)$  for a fixed x. It is easy to show that the function  $f_{\omega}(x)$  is continuous at all real x, and we have:

$$
\begin{cases} f_{\omega}(x) > 0, & \text{when } x \le 0 \\ f_{\omega}(x) < 0, & \text{when } x \ge 1 \end{cases}
$$
 (70)

which implies that  $f_{\omega}(x)=0$  has at least one real root, and all real roots of this equation lie in the interval [0,1]. From the definition of (69), all real roots of  $f_{\omega}(x)=0$  are valid solutions for the characteristic (68) of  $\pi_R$ . In this appendix, we first enumerate the number of roots of  $f_{\omega}(x)=0$  for a given maximum waiting time  $\omega$ , then we prove that the only real root of  $f_{\omega}(x)=0$  must lie in the interval  $[0, -(W_0(\alpha)/1 W_0(\alpha)$ ] when  $\omega > \omega_{-1}$ .

# *A. Determine the Number of Roots of*  $f_{\omega}(x)=0$

For a fixed  $\omega$ , taking the derivative of (69) with respect to x, we have:

$$
f'_{\omega}(x) = \frac{\partial F(x,\omega)}{\partial x} = -h - \left(1 - \frac{2LNx}{\omega}\right)e^{-\frac{2LNx}{\omega}x}.
$$
 (71)

Solving  $f'_{\omega}(x) = 0$ , we obtain the following two roots:

$$
x_0 = \frac{\omega \left[1 - W_0(\alpha)\right]}{2LN},\tag{72}
$$

$$
x_1 = \frac{\omega [1 - W_{-1}(\alpha)]}{2LN},
$$
\n(73)

where  $\alpha = -eh$  is a constant, and  $W_0(\alpha)$  and  $W_{-1}(\alpha)$  are the two possible values of the Lambert  $W$  functions [14] as described in Appendix A. Substituting (72) and (73) into (69), we obtain the following two possible extreme values of  $f_{\omega}(x)$ :

$$
f_{\omega}(x_0) = h + \frac{\omega h \left[1 - W_0(\alpha)\right]^2}{2LNW_0(\alpha)} = h \left(1 - \frac{\omega}{\omega_0}\right),\tag{74}
$$

$$
f_{\omega}(x_1) = h + \frac{\omega h \left[1 - W_{-1}(\alpha)\right]^2}{2LNW_{-1}(\alpha)} = h\left(1 - \frac{\omega}{\omega_{-1}}\right),\tag{75}
$$

where  $\omega_0$  and  $\omega_{-1}$  are parameters defined by (29) and (30), respectively.

Since we know from (70) that all real roots of  $f_{\omega}(x)=0$ lie in the interval [0,1], the function  $f_{\omega}(x)$  exhibits the following properties in the three sub-intervals [0,  $x_0$ ], [ $x_0, x_1$ ], and  $|x_1, 1|$ :

- 1) In  $[0, x_0]$ ,  $f'_{\omega}(x) < 0$  and  $f_{\omega}(x)$  is monotonically decreasing;
- 2) In  $[x_0, x_1]$ ,  $f'_{\omega}(x) > 0$  and  $f_{\omega}(x)$  is monotonically increasing;



Fig. 12. (a) The extreme value of  $f'_{\omega}(x)$  and (b) the two extreme values of  $f_{\omega}(x)$ .



Fig. 13. The function  $f_{\omega}(x)$  for different values of  $\omega$ .

3) In  $[x_1, 1]$ ,  $f'_{\omega}(x) < 0$  and  $f_{\omega}(x)$  is monotonically decreasing again.

It follows that (74) and (75) are the local minima and local maxima, respectively, of the function  $f_{\omega}(x)$ , as illustrated in Fig. 12(b).

TABLE I THE NUMBER OF ROOTS OF THE EQUATION  $f_{\omega}(x)=0$ 

Extreme Values of $f_{\omega}(x)$	Maximum waiting time	Monotonically decreasing	Monotonically increasing	Monotonically decreasing
	ω	interval $[0, x_0]$	interval $[x_0, x_1]$	interval $[x_1, 1]$
$0 < f_{\omega}(x_0) < f_{\omega}(x_1)$	$\omega < \omega_0 < \omega_{-1}$			one root
$0 = f_{\omega}(x_0) < f_{\omega}(x_1)$	$\omega = \omega_0 < \omega_{-1}$	$x_0$	$x_0$	one root
$f_{\omega}(x_0) < 0 < f_{\omega}(x_1)$	$\omega_0 < \omega < \omega_{-1}$	one root	one root	one root
$f_{\omega}(x_0) < f_{\omega}(x_1) = 0$	$\omega_0 < \omega = \omega_{-1}$	one root	$x_1$	$x_1$
$f_{\omega}(x_0) < f_{\omega}(x_1) < 0$	$\omega_0 < \omega_{-1} < \omega$	one root		

The number of roots of the function  $f_{\omega}(x)$  is determined by the two extreme values  $f_{\omega}(x_0)$  and  $f_{\omega}(x_1)$  given by (74) and (75), respectively. Since  $f_{\omega}(x)$  is monotonically increasing in the interval  $[x_0, x_1]$ , we know that  $f_\omega(x_0) < f_\omega(x_1)$ , or equivalently  $\omega_0 < \omega_{-1}$ , always holds. For different values of the maximum waiting time  $\omega$ , as illustrated in Fig. 13, the number of solutions of the characteristic (68) for different values of  $\omega$  is summarized in Table I.

#### *B. Proof of Corollary 2*

Suppose that  $\omega_0 < \omega_{-1} < \omega$ . We know that the function  $f_{\omega}(x)=0$  has a unique real root in the interval  $[0, x_0]$ , where  $f'_{\omega}(x_0) = 0$  and  $f_{\omega}(x)$  has a local minima at  $x_0$ . We want to show that this root of  $f_{\omega}(x)=0$  lies in the interval  $[0,-(W_0(\alpha)/1-W_0(\alpha))].$ 

Recall that  $f_{\omega}(0) > 0$  for all  $\omega$ , and  $f_{\omega}(x)$  has a local minima at:

$$
x_0 = \frac{\omega \left[1 - W_0(\alpha)\right]}{2LN}.\tag{76}
$$

When  $\omega = \omega_0$ , denote this local minima as  $x_0 = a$  and  $f_{\omega_0}(a)=0$ . From (29), we have:

$$
a = \frac{\omega_0 \left[1 - W_0(\alpha)\right]}{2LN} = -\frac{W_0(\alpha)}{1 - W_0(\alpha)} > 0. \tag{77}
$$

For a fixed x, taking the derivative of (69) with respect to  $\omega$ , we obtain:

$$
g'_x(\omega) = \frac{\partial F(x,\omega)}{\partial \omega} = -\frac{2LNx^2}{\omega^2} e^{-\frac{2LN}{\omega}x} < 0. \tag{78}
$$

Thus, the function  $g_x(\omega)$  is monotonically decreasing with respect to  $\omega$ , for any fixed x. In particular, when  $\omega_0 < \omega_{-1} < \omega$ , we have:

$$
g_{x=a}(\omega) < g_{x=a}(\omega_0) = F(a, \omega_0) = f_{\omega_0}(a) = 0,\tag{79}
$$

which, however, implies:

$$
f_{\omega}(a) = F(a, \omega) = g_{x=a}(\omega) < 0. \tag{80}
$$

Since we know  $f_{\omega}(0) > 0$ , it follows from (80) that the unique real root of  $f_{\omega}(x)=0$  must lie in the interval  $[0, a] =$  $[0, -(W_0(\alpha)/(1 - W_0(\alpha))],$  as illustrated in Fig. 13.

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