Add/Drop Flexibility and System Complexity Tradeoff in ROADM Designs

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Abstract—As a key component in dynamic wavelength-routing optical networks (WRONs), the contention performance at add/drop (A/D) ports of ROADMs has attracted a lot of attention in recent years. For the first time, this paper derives the close-form solutions of the blocking probability (BP) of A/D requests to characterize the fundamental tradeoff between A/D flexibility and system complexity in the ROADM. We show that the ROADM with fiber cross-connect (FXC) can decide whether a request can be satisfied based on global transceiver-usage information, and thus an exact expression of BP can be obtained. In comparison, the ROADM without FXC needs detail transceiver-usage information to make decision, and thus is hard to be analyzed. To circumvent the difficulty in analysis, we describe the evolution of the number of busy transceivers using a one-dimensional Markov chain, where the state transition rates are estimated from global transceiver-usage information. Based on this model, we obtain an approximate BP the accuracy of which is high enough for currently commercialized ROADMs and increases with the number of drop ports. Our analytical results provide an interesting insight into the tradeoff between A/D flexibility and system complexity, based on which we give some suggestions for system design.

Index Terms—Reconfigurable optical add/drop multiplexer (ROADM), contentionless, blocking probability, modeling

I. INTRODUCTION

Reconfigurable optical add/drop multiplexer (ROADM) is a key component of dynamic wavelength-routing optical networks (WRONs). According to traffic variation, ROADMs enable the WRON to provide dynamic mesh interconnection for network nodes without manual intervention [1]. Owing to flexibility and efficiency, ROADMs have been widely deployed in backbone, metro, and regional WRONs [2]–[4].

As Fig. 1 plots, a ROADM node mainly includes an optical switching core, a number of inbound/outbound degrees, and a set of add/drop (A/D) ports. The ROADM connects to remote nodes via inbound/outbound degrees, and to a local electrical switch via transmitters/receivers (Txs/Rxs) at the A/D ports. Via the configuration of optical switching core, the ROADM can switch a wavelength from an inbound degree to an outbound degree, such that a lightpath, e.g., LP1 in Fig. 1, can bypass this node without optical-electrical-optical (o-e-o) conversion. In the same way, the ROADM can also connect an A/D port with an outbound/inbound degree to add/drop a wavelength, e.g., LPs 2 and 3 in Fig. 1.

One of important issues in the design of ROADMs is A/D flexibility, which is featured as colorless, directionless, and contentionless [5]–[8]. The ROADM with first two features is called CD-ROADM [5]–[8], and that with three features is referred to as CDC-ROADM. The CDC-ROADM possesses the highest A/D flexibility [9]. However, contentionless property comes with high system complexity, as Fig. 1 illustrates. In Fig. 1(a), each drop port contains 3 Rxs, and one Rx is free in drop port 3. In this case, a request that needs to drop the red wavelength at inbound degree 2 (dotted line in Fig. 1(a)) will be blocked by the existing red lightpath in drop port 2 due to wavelength contention, which occurs when two lightpaths at the same wavelength compete for the same A/D port. One way to remove the wavelength contention at drop ports is to increase the number of ports such that each port only contains one Rx, as Fig. 1(b) plots. In this case, the request can be served. However, this remarkably increases the port count of the optical switching core, meaning that the system complexity increases. Therefore, there is a fundamental tradeoff between A/D flexibility and system complexity.
A. Previous Works

To facilitate system design, a number of papers have been focused on the performance evaluation of ROADMs in terms of the blocking probability (BP) of A/D requests through simulations or analytical methods.

Most of previous works carry out study via simulation. Ref. [10] compares the BP of different combinations of colorless, directionless and contentionless features, and concludes that the BP of CD-ROADM will converge when the number of A/D ports is larger than a threshold. Ref. [11] studies the value proposition of contentionless feature, under the condition that the traffic dynamicity of network is limited. Simulation results indicate that the performance enhancement brought by contentionless feature is only visible under heavy traffic load. To alleviate wavelength contention at the A/D ports, Ref. [12] proposes a CD-ROADM with a client-side fiber cross-connect (C-FXC), which is employed to enable resource pooling such that the Tx’s/Rx’s can be shared by all the A/D ports. Refs. [8], [13] compare the intra-node contention performance of the CD-ROADMs with and without C-FXC, which shows that the C-FXC can improve the performance significantly. For a ROADM with C-FXC, increasing A/D ports can cut down the BP when the number of A/D ports is less than the nodal degree. However, the introduction of C-FXC will increase the system cost remarkably. For this reason, Ref. [14] proposes to replace the single large-scale C-FXC by a set of multiple small C-FXC modules and demonstrates that the replacement does not incur a remarkable performance loss. Another way to reduce system cost is to employ partially contentionless CD-ROADM (CDpC-ROADM) [7], where a part of A/D ports are CDC ports and the others are conventional. The simulation results show that the CDpC-ROADM can perform similarly to a CDC-ROADM when the ratio of CDC A/D ports reaches 60%. Refs. [2], [15]–[17] propose to take the wavelength contention in CD-ROADM into consideration in routing and wavelength assignment (RWA) algorithms used by WRONs. However, as Refs. [8], [13] point out, this scheme is undesirable since it would add complexity to the RWA problem.

However, simulations are typically time consuming [18], [19], especially when to observe the blocking performance of a lightly loaded system. In contrast, analytical approaches can not only obtain the results faster but also provide better insight into system features, which would be a useful guide for the service provider to design WRON [20].

Thus, Refs. [5], [19], [21] seek to investigate the blocking performance of ROADMs using theoretical methods. Ref. [19] proposes an analytical model for the ROADM with a C-FXC. This model first treats the nodal degrees and the A/D ports as the virtual links of a two-hop optical path, and then applies the method developed for the analysis of lightpath blocking performance of WRONs [22] to study the intra-node contention of ROADMs. Under the assumption that the traffic load offered to each A/D port is independent of each other, this model calculates the approximate BP in an iterative manner. The numerical results in [19] indicate that a CD-ROADM with a small number of A/D ports can achieve a BP almost the same with that of a CDC-ROADM. Refs. [5], [21] extend this model to evaluate the blocking performance of the WRON equipped with CD-ROADM and draw a similar conclusion.

B. Our Works

This paper focuses on the analysis of intra-node wavelength contention in the ROADM with and without FXC. Our goal is to obtain closed-form expressions of BP, which can not only predict the performance of currently commercialized ROADMs accurately and quickly but also provide a good insight into performance tradeoff.

We first analyze the performance of ROADMs with FXC. Since the ROADM with FXC can decide whether a request can be satisfied based on global Rx-usage information, we show that it can be modelled as a multi-dimensional Markov chain, the steady-state probability of which has a product-form expression. We obtain the exact solutions of BP. By comparison, the analysis of the ROADM without FXC is much more difficult, since it needs detail Rx-usage information to make decision. To circumvent the difficulty in analysis, we describe the evolution of the number of busy Rxs using a one-dimensional Markov chain, the state transition rate of which is amended by a conditional blocking probability estimated from global Rx-usage information. We obtain an approximate BP, the accuracy of which is high for currently commercialized ROADMs and increases with the number of drop ports.

Using the analytical results, we study the contention performance of ROADMs. We have the interesting findings as follows.

(f1) Wavelength contention and Rx-resource exhaustion dominate the BP when the traffic load is relatively low and relatively high, respectively.

(f2) Given the number of A/D ports, the traffic load region where wavelength contention is the dominant factor of BP becomes wider when the number of Rxs is larger.

(f3) Installing FXC or increasing the number of A/D ports can reduce the BP incurred by wavelength contention.

According to these findings, we suggest that:

(s1) For a ROADM with a small number of Rxs, $H = D$ A/D ports are enough to achieve the best performance and it is unnecessary to install the FXC, where $H$ is the number of A/D ports and $D$ is the degree of the ROADM.

(s2) For a ROADM with rich Rx-resource, installing an FXC or $H > D$ drop ports is helpful to reducing the BP when the traffic load is not heavy.

The rest of our paper is organized as follows. Section II briefly introduces the structure of the ROADMs with and without FXC to facilitate our analysis. We first carry out exact analysis for the ROADM with FXC in Section III, and then approximate estimation for the ROADM without FXC in Section IV. Meanwhile, we investigate the tradeoff between A/D flexibility and system complexity using our analysis results. We conclude this paper in Section V.
II. PRELIMINARY

To facilitate the analysis, we introduce the structure and the function of ROADMs in this section. In particular, we briefly describe the ROADMs with and without FXC in Section II-A and B, respectively, and then we present the assumptions for our analysis in Section II-C.

A. ROADM with FXC

Fig. 2 plots a ROADM with FXC. The ROADM connects to its neighbor nodes via \( D \) inbound/outbound degrees, each of which carries \( W \) wavelengths, and with local electrical switches via \( H \) A/D ports attached to the FXC. Let \( R \) be the total number of Txs/Rxs in a ROADM. The ROADM in Fig. 2 is a \( 4 \times 4 \) ROADM, where \( D = 4, W = 4, H = 2, \) and \( R = 6. \)

The inbound/outbound degrees and A/D ports are connected via a \((D + H) \times (D + H)\) optical switching core. There are multiple ways to implement optical switching core. For example, the switching core could be constructed from \( 2(D + H) \) wavelength sensitive switches (WSSs) [23], [24] or the combination of \( D + H \) \((D + H)\) splitters and \( D + H \) \((D + H)\) WSSs [6], [12].

At the drop side, there are \( H \times W \) wavelength demultiplexers (DeMux), each of which is associated with a drop port, an \( H \times W \) FXC, and \( R \) Rxs. The DeMux demultiplexes the multiple-wavelength signal at each drop port. The function of a DeMux could also be implemented by a WSS. The FXC performs optical switching between the outputs of \( H \times W \) DeMuxs and \( R \) Rxs, as Fig. 2 exhibits. Thanks to the FXC, different drop ports can share all the \( R \) Rxs. For example, all the idle Rxs are available for the lightpath at drop port 2 in Fig. 2. Due to wavelength contention, two lightpaths at the same wavelength cannot be dropped via the same drop port. Even so, the two lightpaths at red wavelength in Fig. 2 can be dropped via ports 1 and 2, respectively. Clearly, when \( H = D \), with the help of FXC, up to \( D \) lightpaths at a wavelength can be dropped via \( D \) drop ports simultaneously. Thus, the ROADM with FXC and \( H = D \) is a CDC-ROADM.

We refer to a request that requires to drop a lightpath at wavelength \( \lambda_i \) as \( \lambda_i \)-request, and a lightpath at \( \lambda_i \) that is dropped at this node as \( \lambda_i \)-lightpath, where \( i = 1, 2, \cdots, W. \) Let \( n_i \) be the number of \( \lambda_i \)-lightpaths. According to the above descriptions, a \( \lambda_i \)-request can be accepted, if

A1. There is at least one drop port that does not carry a \( \lambda_i \)-lightpath, i.e., \( n_i \leq H - 1; \)
A2. There is at least one idle Rx in the ROADM, i.e., \( \sum_{i=1}^{W} n_i \leq R - 1. \)

We refer to \( n_1, n_2, \cdots, n_W \) as global Rx-usage information. Conditions A1 and A2 imply that the ROADM only needs to know the global Rx-usage information to judge whether it can satisfy a request.

Note that the procedure of lightpath adding is quite similar to that of lightpath dropping. Thus, this paper only focuses on the analysis of the BP of drop requests.

B. ROADM without FXC

Fig. 1 shows a ROADM without FXC, where each A/D port connects with \( R/H \) Txs/Rxs directly via a \( 1 \times R/H \) WSS. To facilitate the discussion, we only consider the case where \( R \) is an integer multiple of \( H \) in this paper.

Since there is no FXC, each drop port is only associated with \( R/H \) Rxs. For example, the request at red wavelength, which is represented by a dotted line in Fig. 1, cannot use the first two Rxs via drop port 2, since they are dedicated to drop port 1. As a result, this request will be blocked. Clearly, the ROADM is contentionless only when \( H = R. \)

Let \( b_{i,j} \) be the number of \( \lambda_i \)-lightpaths activate at drop port \( b, \) where \( i = 1, 2, \cdots, W \) and \( b = 1, 2, \cdots, H. \) Consider a \( \lambda_i \)-request. We call a drop port an available port if it satisfies the following conditions:

B1. Drop port \( b \) does not carry \( \lambda_i \)-lightpaths, i.e., \( n_{b,i} = 0; \)
B2. More than one Rx is idle at drop port \( b, \) i.e., \( \sum_{j \neq i} n_{b,j} \leq R/H - 1. \)

The \( \lambda_i \)-request can be accepted, if there is more than one available port in the system. In this case, the ROADM randomly selects an available port to the request.

We refer to conditions B1 and B2 as detail Rx-usage information. As Section IV will present, the need of detail Rx-usage information makes the analysis of ROADMs without FXC much more difficult than that of ROADM with FXC.

C. Assumptions in Analysis

To facilitate our analysis, we make the following assumptions:

(a1) All the wavelength channels are statistically identical.
(a2) When a wavelength at an inbound degree is idle, the arrival of drop-requests at this wavelength follows a Poisson process with rate \( \gamma. \) Herein, a wavelength channel is idle if it is not occupied by a lightpath terminating at this node.
(a3) If the ROADM can accept a request, it will assign an available drop port to the request in a random manner.
(a4) The holding time of a lightpath is an exponential random variable with parameter \( \mu. \)
(a5) The number of Rxs is larger than the number of degrees and less than the product of the number of degrees and the number of wavelengths, i.e., \( D < R < DW. \)

Thus, the traffic load per wavelength channel is \( \rho = \gamma/\mu. \)
III. THE ROADM WITH FXC

In this section, we evaluate the BP of the ROADM with FXC. In Section III-A, we show that the system can be modelled as a Markov chain and derive the stationary state probabilities. Based on this model, Section III-B obtains the BP, which reveals the system properties in Section III-C.

A. System Model

Recall that the ROADM with FXC should know the global Rx-usage information to figure out whether a request can be satisfied. We thus use a vector

\[ s = (n_1, n_2, \cdots, n_W) \]

to delineate the state of the ROADM with FXC, where \( n_i \geq 0 \) is the number of \( \lambda_i \)-lightpaths and \( i = 1, 2, \cdots, W \).

State \( s \) may transit to other states. When a \( \lambda_i \)-request arrives and conditions A1 and A2 hold, the ROADM will accept this request, and \( s \) moves to

\[ s_i^+ = (n_1, n_2, \cdots, n_i + 1, \cdots, n_W). \]

When a \( \lambda_i \)-lightpath is torn down, \( n_i \) decreases by 1 and \( s \) changes to

\[ s_i^- = (n_1, n_2, \cdots, n_i - 1, \cdots, n_W). \]

It is obvious that the state transition from state \( s \) is irrelevant to the states, from which the system transited to state \( s \). We thus model the Rx occupancy in the ROADM with FXC as a \( W \)-dimensional continuous-time Markov chain.

The state transition rates are determined as follows. Consider state \( s \), where there are \( n_i \) active \( \lambda_i \)-lightpaths and \( D - n_i \) free channels at wavelength \( \lambda_i \) at the inbound degrees. In this case, a \( \lambda_i \)-request will arrive at the ROADM with rate \( \gamma(D - n_i) \). It follows that the transition rate from \( s \) to \( s_{i(i^+)} \) is \( \gamma(D - n_i) \) as long as \( n_i \leq H - 1 \) and \( \sum_{i=1}^{W} n_i \leq R - 1 \). On the other hand, a \( \lambda_i \)-lightpath is torn down with rate \( \mu \), and thus the transition rate from \( s \) to \( s_{i(i^-)} \) is \( \mu n_i \).

Fig. 3 plots the Markov chain for a ROADM with FXC, where \( D = 4, W = 2, R = 4, \) and \( H = 3 \). It is very easy to show that the transition rate between \( s \) and \( s_{i(i^+)} \) is completely decided by \( n_i \) and is unrelated to \( n_j \) for all \( j \neq i \), as long as \( n_i \leq H - 1 \) and \( \sum_{i=1}^{W} n_i \leq R - 1 \). For example, in Fig. 3, the transition rate from state (2,1) to state (3,1) and that from state (2,1) to state (1,1) only depends on \( n_1 = 2 \). Let \( \pi_s \) be the stationary probability of state \( s \). The above observation hints that \( \pi_s \) may satisfy a set of detailed balance equations and thus have a solution in product form [27], which is formally stated by the following theorem.

**Theorem 1:** Stationary probability of state \( s \) of the ROADM with FXC is given by:

\[ \pi_s = \frac{\prod_{i=1}^{W} \left[ \frac{(D_{n_i})^n_{\mu n_i}}{G} \right] }{G} \]

where

\[ G = \sum_{n_1=0}^{n_1} \cdots \sum_{n_W=0}^{n_W} \left[ \prod_{i=1}^{W} \left( \frac{(D_{n_i})^n_{\mu n_i}}{G} \right) \right], \]

\[ u_i = \min \left\{ H, R - \sum_{j=1}^{i-1} n_j \right\}, \] and \( 1 \leq i \leq W \).

**Proof:**

\[ (D - n_i) \gamma \pi_s = (D - n_i) \gamma \frac{\prod_{i=1}^{W} \left[ \frac{(D_{n_i})^n_{\mu n_i}}{G} \right] }{G}, \]

\[ = (D - n_i) \mu (D_{n_i})^n_{\mu n_i} \prod_{j \in J} \left[ \frac{(D_{n_j})^n_{\mu n_j}}{G} \right], \]

\[ = (n_i + 1) \mu \prod_{j \in J} \left[ \frac{(D_{n_j})^n_{\mu n_j}}{G} \right]. \]

where \( J = \{ 1, 2, \cdots, i - 1, i + 1, \cdots, W \} \). This derivation clearly indicates that \( \pi_s \) satisfies the detailed balance equations and thus is the stationary probability of state \( s \).

B. Blocking Probability

To facilitate our presentation, we derive the BP of requests by calculating the BP of \( \lambda_1 \)-requests, as we assume that all the wavelength channels are statistically identical in assumption (a1). The BP, denoted by \( P_B \), is the ratio of the number of blocked requests to the number of requests that arrive at the ROADM. Note that the steady-state probability \( \pi_s \) is the time-average probability of state \( s \). To obtain the BP, it is necessary to derive the probability that a request sees the ROADM staying at state \( s \) from \( \pi_s \).

Consider a long time period \([0, T]\). The number of \( \lambda_1 \)-requests that arrive when the ROADM stays at state \( s \) is given by \( N_s = T \times \pi_s \times (D - n_1) \gamma \). Let \( S \) be the set of all the states, and \( \pi_s^* \) be the probability that a \( \lambda_1 \)-request sees the ROADM staying at state \( s \). Using Eq.(1), \( \pi_s^* \) can be calculated as follows

\[ \pi_s^* = \frac{N_s}{\sum_{s \in S} N_s} = \frac{(D_{n_1} - 1)^n_{\mu n_1} \prod_{i=2}^{W} \left[ \frac{(D_{n_i})^n_{\mu n_i}}{G} \right] }{G}, \]

Fig. 3. Markov chain of a ROADM with FXC, where \( D = 4, W = 2, R = 4, \) and \( H = 3 \).
where

\[ G^* = \sum_{n_1=0}^{u_1} \cdots \sum_{n_w=0}^{u_W} \left\{ \binom{D-1}{n_1} \rho^{n_1} \prod_{i=2}^{W} \left[ \binom{D}{n_i} \rho^{n_i} \right] \right\}. \]

As Section II-B presents, the blocking of \( \lambda_1 \)-requests can be caused by the following two reasons.

Type-1 blocking: Rxs are all busy, i.e., \( \sum_{i=1}^{W} n_i = R \).

Type-2 blocking: The number of \( \lambda_1 \)-lightpaths is \( H \) while Rxs are not all busy, i.e., \( \sum_{i=1}^{W} n_i < R \) and \( n_1 = H \).

Type-1 blocking and type-2 blocking are mutually exclusive.

Type-1 blocking is induced by the states in the set \( S_1 = \{ s \mid \sum_i n_i = R \} \). For example, type-1 blocking will happen if a request sees states (3,1), (2,2), and (1,3) in Fig. 3. Thus, \( P_1 \) can be determined by \( P_1 = \sum_{s \in S_1} P_s^* \). Enumerating the states in \( S_1 \), we can obtain \( P_1 \) as follows:

\[ P_1 = \frac{\rho^R}{G} \sum_{n_1=1}^{U_1} \cdots \sum_{n_w=1}^{U_W} \left\{ \binom{D-1}{n_1} \right\} \]

\[ \times \left( R - \sum_{j=1}^{W} \frac{D}{D} n_j \right) \prod_{i=2}^{W-1} \left( \frac{D}{D} n_i \right), \]

where the limits of the \( w \)-th summation

\[ l_w = \max \left\{ 0, R - \sum_{j=1}^{w-1} n_j - (W - w) H \right\} \]

and

\[ v_w = \min \left\{ H, R - \sum_{j=1}^{w-1} n_j \right\} \]

are determined from the constraints \( \sum_{i=1}^{W} n_i = R \) and \( n_w \leq H \), and \( w = 1, 2, \ldots, W - 1 \).

Type-2 blocking occurs when the \( \lambda_1 \)-request sees the states in the set \( S_2 = \{ s \mid \sum_{i=1}^{W} n_i < R \) and \( n_1 = H \} \). As an instance, in Fig. 3, a request will be rejected by type-2 blocking, if it sees state (3,0). We thus have \( P_2 \) as follows:

\[ P_2 = \frac{\binom{D-1}{H}}{G^*} \sum_{n_2=1}^{y_2} \cdots \sum_{n_W=1}^{y_W} \left\{ \prod_{i=2}^{W} \binom{D}{n_i} \right\} \rho^R H \sum_{j=2}^{w-1} n_j, \]

where

\[ y_w = \min \left\{ H, R - 1 - H - \sum_{j=2}^{w-1} n_j \right\} \]

is determined from the constraints \( \sum_{i=1}^{W} n_i < R \) and \( n_1 = H \).

Combining (3) and (4), we have the following corollary.

**Corollary 1**: The BP of requests in a ROADM with FXC is given by:

\[ P_B = P_1 + P_2 \]

\[ (5) \]

### C. Numerical Results and Discussions

We study the blocking performance of ROADMs with FXC in this part, using the analytical results in Section III-B. To verify the accuracy of our model, we compare our theoretical results with simulation results. To simulate the steady-state performance, we use moving-window based method [28] in our simulation. We update the BP each time when there is a blocked request. We compute the mean and the variance of the last 100 records of BP, and stop the simulation when the coefficient of variation [29] is less than 10^{-4}. Since the nodal degree in a real network is typically 3 through 5, and less than 8 [30], we consider a ROADM with FXC, where there are \( D = 8 \) degrees and each degree carries \( W = 40 \) wavelengths, as an example in our study. We set \( R = 80 \) and \( R = 160 \), as the number of Rxs in a ROADM is typically less than 50% of \( DW \) in practice.

Fig. 4 plots type-1 BP \( P_1 \), type-2 BP \( P_2 \), and BP \( P_B \) changing with traffic load \( \rho \), where the number of drop ports \( H = 6 \). When \( \rho \) is low, though the Rxs are unlikely exhausted, blocking could still occur due to wavelength contention. For example, a \( \lambda_1 \)-request will be blocked if there happens to be 6 \( \lambda_1 \)-lightpaths at the drop ports. Thus, \( P_B \) is almost determined by \( P_2 \) when \( \rho \) is low in Fig. 4. When \( \rho \) is heavy, almost all the Rxs are occupied by the lightpaths. Though a request may be blocked by wavelength contention in this case, it will most likely be rejected by Rx-resource exhaustion. As Fig. 4 shows, \( P_2 \) slightly decreases, whereas \( P_1 \) increases very fast and almost determines \( P_B \) when \( \rho \) is high. Thus, from Fig. 4, we have the first finding as follows:

(f1) Wavelength contention and Rx-resource shortage determine the BP when \( \rho \) is low and high, respectively.

Presumably, wavelength contention has more impacts on system performance if the ROADM is equipped with more Rxs. If the ROADM has more Rxs, it can support a higher traffic load before the Rx-resource becomes insufficient. Thus, for the ROADM with larger \( R \), type-2 blocking could have a significant impact on the BP in a wider traffic-load region. This point is confirmed by the comparison of Fig. 4(a) and (b). Type-2 BPs in Fig. 4(a) and 4(b) are almost the same when \( \rho \) is smaller than 0.2. However, type-2 BP in Fig. 4(a) only increases slightly and then declines when \( \rho \) increases from 0.2 to 1, while that in Fig. 4(b) increases to ~0.06 when \( \rho \) is
and system complexity. Let \( P_B \) be the BP of a ROADM if the number of drop ports is \( H \). We define \( H \) as the saturation point of BP if \( P_B^{H+1}/P_B^H > 95\% \) and \( g = P_B^{H+1}/P_B^H \) as performance gain in BP by increasing \( H \) from \( R/W \) to \( D \).

Fig. 5 plots the BP as a function of \( H \) under different \( R/W \), where \( H \) increases from \( R/W \) to \( D = 8 \). As expected, the BP falls down with the increase of \( H \), which clearly demonstrates the tradeoff between A/D flexibility and system complexity. In addition, Fig. 5 shows more interesting results. In Fig. 5(a) where \( R = 80 \), the BP at \( \rho = 0.75 \) saturates at \( H = 3 \) and \( g \) is only 1.16, while the BP at \( \rho = 0.25 \) saturates at \( H = 6 \) and \( g \) climbs up to 714.29. Also, comparing Fig. 5(b) with (a), we find that \( g \) at each \( \rho \) increases remarkably when \( R \) is doubled. In particular, the BPs at different \( \rho \)s in Fig. 5(b) keep declining and do not saturate until \( H = D = 8 \). Recall that wavelength contention is the dominant factor of BP when \( R \) is small and \( \rho \) is low or when \( R \) is large and \( \rho \) is not high, as Fig. 4 exhibits. From Fig. 5, we can obtain the third finding as follows.

(f1) \textbf{Increasing the number of drop ports (i.e., the port count of FXC) can reduce the the BP incurred by wavelength contention.}

In addition to providing physical insight into the system, our model can obtain accurate performance predictions for ROADMs with FXC much faster than simulation. Both Fig. 4 and 5 confirm the accuracy of our analytical result. Fig. 6 further compares the time needed by simulation and analytical model to obtain the BP. We perform the simulation on C++ platform, and calculate the analytical result on Matlab platform. Fig. 6 shows that the time needed by simulation is inversely proportional to the BP. For example, it takes more than 2.5 hours to simulate the ROADM with a BP of \( 10^{-6} \). As a comparison, the time required by the calculation of (5) is only \( \sim 0.2 \text{ms} \), and almost keeps unchanged for all the BPs. This indicates that our result in (5) can be used to predict the performance of ROADMs during system design.

IV. THE ROADM WITHOUT FXC

As Section II explains, the ROADM without FXC needs more information than the one with FXC to decide whether it is able to accept a request, which makes the analysis intractable. We thus propose an approximate model in Section IV-A to analyze the ROADM without FXC, and drive an approximate expression of BP in Section IV-B. Section IV-C shows that our analytical result is accurate enough for commercial ROADMs to provide a good insight into system features.

A. Approximate System Model

To figure out whether there is a port available to a request, the ROADM has to possess the detail Rx-usage information of each drop port, which includes not only the wavelength of each lightpath that occupies the Rxs (constraint B1), but also the number of busy Rxs (constraint B2). Recall that \( n_{i,j} = 0, 1 \) is the number of Rxs occupied by wavelength \( \lambda_j \) in drop port \( i \), where \( i = 1, 2, \cdots, H \) and \( j = 1, 2, \cdots, W \). If there is no \( \lambda_j \)-lightpath dropped via port \( i \), \( n_{i,j} = 0 \); otherwise, \( n_{i,j} = 1 \). The ROADM should keep the state information defined by an \( HW \)-dimensional vector

\[
\hat{s} = (n_{1,1}, \cdots, n_{1,W}; n_{2,1}, \cdots, n_{2,W}; \cdots; n_{H,1}, \cdots, n_{H,W})
\]

which is also called an Rx-usage pattern in this paper. It is easy to show the traffic drop process of ROADM without FXC can be described by an \( HW \)-dimensional Markov chain, the state of which is defined by vector \( \hat{s} \). However, the transition from state \( \hat{s} \) to other states is very complex. The transition rate from state \( \hat{s} \) with \( n_{i,j} = 0 \) to state \( \hat{s}' \) with \( n_{i,j} = 1 \) depends on all the components in vector \( \hat{s} \), while that from \( \hat{s}' \) to \( \hat{s} \) is a constant \( \mu \). This indicates that the detailed balance equations do not hold in this case, which makes the analysis of \( HW \)-dimensional Markov chain very hard [27].

In this part, we pursue approximate analysis with a high degree of approximation. Our key idea is to estimate the impact of wavelength contention from the total number of busy Rxs, a kind of global Rx-usage information, to circumvent the difficulty in exact analysis. In particular, we describe the evolution of the total number of busy Rxs in the ROADM using a one-dimensional Markov chain, as Fig. 7 plots, where the state transition rate is amended by an estimated blocking probability under the condition that \( r \) Rxs are busy in the ROADM. In the
we associate both the wavelengths and the number of busy Rxs in each port. Thus, to enumerate all possible patterns, we need to enumerate the number of busy Rxs in each port under each association. Let \( r_i \) be the number of busy Rxs in port \( i \). The number of selections of \( r_i \) wavelengths from \( R/H \) wavelengths is \((r_i/H)\). Note that, given a port-to-wavelength association, the wavelength selection in a port is irrelevant to that in other ports. Thus, the number of Rx-usage patterns under an association can be calculated as follows:

\[
\sum_{r_1=L_1}^{U_1} \cdots \sum_{r_H=L_H}^{U_H} \prod_{i=1}^{H} \binom{R/H}{r_i} = \binom{R}{r},
\]

where the limits

\[
U_i = \min \{R/H, r - \sum_{j=1}^{i-1} r_j \}
\]

and

\[
L_i = \max \{0, r - \sum_{j=1}^{i-1} r_j - (H-i)R/H \}
\]

are determined according to the constraint \( \sum_{i=1}^{H} r_i = r \). Fig. 8(b) plots two Rx-usage patterns under the association in Fig. 8(a), where each grey circle represents a busy Rx. Rx-usage patterns 1 and 2 are corresponding to \( s_1 = (1, 1, 0; 0, 1, 0) \) and \( s_2 = (1, 0, 0; 0, 1, 1) \). Let nature number \( A \) be the number of associations. The total number of Rx-usage patterns equals to

\[
A_r = A \binom{R}{r}.
\]

When deriving \( B_r \), we focus on the enumeration of the patterns that block requests. A request will be blocked if no drop port is available, that is, each port either carries a lightpath at the wavelength of request or has no free Rxs. Suppose \( h \) of \( H \) ports, marked \( z_1, z_2, \ldots, z_h \), have free Rxs, and these ports have \( r_{z_1}, r_{z_2}, \ldots, r_{z_h} \) busy Rxs, respectively.
under an association. We only consider the cases where \( h < D \). Otherwise, a request will not be blocked definitely, since the number of drop lightpaths at the wavelength of this request is not more than \( D - 1 \) and thus one of the \( h \) ports must be available. For \( z_i \), there are \( (\frac{R}{h})^{w} \) possible Rx-usage patterns under an association. Also, \( z_i \) is unavailable if it carries a lightpath at the wavelength of request. It follows that port \( z_i \) is unavailable with probability \( r_{z_i}/W \). Thus, the number of Rx-usage patterns of port \( z_i \) that are unavailable for a request is \( (\frac{R}{h})^{w} r_{z_i}/W \). As a result, under an association, the number of Rx-usage patterns that all \( h \) ports are unavailable is 

\[
K \approx \sum_{r_{z_1}=l_1}^{u_1} \cdots \sum_{r_{z_h}=l_h}^{u_h} \prod_{i=1}^{h} \left( \frac{R}{h} \right) r_{z_i}/W
\]

according to approximation (B*), where the limits

\[
u_i = \min \left\{ \frac{R}{h} - 1, r - \sum_{j=1}^{i-1} r_{z_j} - (H - h) R/H \right\}
\]

and

\[
l_i = \max \left\{ 0, r - \sum_{j=1}^{i-1} r_{z_j} - (H - h) R/H - (h - i) \left( \frac{R}{H} - 1 \right) \right\}
\]

are determined according to the condition that each of \( h \) ports has idle Rxs. Noting that the number of selections of \( h \) ports from \( H \) ports is \( \binom{H}{h} \), we can obtain \( B_r \) as follows:

\[
B_r \approx A \sum_{h=1}^{\min\{H,D-1\}} \left\{ \binom{H}{h} K \right\}
\]  \( \tag{10} \)

Substituting (9) and (10) into (8), we have:

\[
p_r \approx \sum_{h=0}^{\min\{H,D-1\}} \left\{ \binom{H}{h} K \right\} / \left( \frac{r}{R} \right)
\]  \( \tag{11} \)

Using (7) and (11), we derive the approximate stationary probability of state \( r \) as follows:

\[
\hat{\pi}_r \approx \frac{(\frac{D}{R})^r \prod_{k=0}^{r-1} (1 - p_k)}{\sum_{r'=0}^{R} \left( (\frac{D}{R})^{r'} \prod_{k=0}^{r'-1} (1 - p_k) \right)}
\]  \( \tag{12} \)

**B. Approximate Blocking Probability**

Let \( \hat{P}_B \) be the BP of ROADM without FXC. To derive \( \hat{P}_B \), we need the probability that a request sees \( r \) busy Rxs, which is denoted by \( \hat{\pi}_r^* \). Using the technique presented in Section III-B, we can derive \( \hat{\pi}_r^* \) from \( \pi_r^* \) as follows:

\[
\hat{\pi}_r^* \approx \frac{(\frac{D}{R})^{r-1} \prod_{k=0}^{r-1} (1 - p_k)}{\sum_{r'=0}^{R} \left( (\frac{D}{R})^{r'} \prod_{k=0}^{r'-1} (1 - p_k) \right)}
\]

Let \( \hat{P}_1 \) and \( \hat{P}_2 \) be the probabilities of type-1 blocking and type-2 blocking, respectively. Clearly, \( \hat{P}_1 \approx \hat{\pi}_R^* \) since type-1 blocking is induced by Rx-resource exhaustion, while

\[
\hat{P}_2 \approx \sum_{r=0}^{R-1} \hat{\pi}_r^* p_r,
\]

since type-2 blocking is induced by wavelength contention when \( r < R \).

Hence, the approximate BP of requests in a ROADM without FXC is given by:

\[
\hat{P}_B = \hat{P}_1 + \hat{P}_2 \approx \hat{\pi}_R^* + \sum_{r=0}^{R-1} \hat{\pi}_r^* p_r.
\]  \( \tag{13} \)

Note that Eq. (13) is obtained under approximations (A*) and (B*), which are used to deal with wavelength contention in our analysis. This indicates that Eq. (13) has error when the number of drop ports \( H < R \), that is, when there is wavelength contention in the ROADM. It is expected that, with the increase of \( H \), wavelength contention is gradually weakened and thus the error reduces. Especially, when \( H = R \), the ROADM is a CDC-ROADM and there is no wavelength contention. In this case, \( R/H = 1 \) and \( u_i = 0 \) in (11) for all \( i = 1, 2, \ldots, h \), and thus \( p_r = 0 \) for all \( r = 0, 1, \ldots, R - 1 \). As a result, Eq. (13) changes to the equation as follows

\[
\hat{P}_B = \hat{\pi}_R^* = \prod_{r=0}^{(W/D-1)} \frac{(R/W)^r}{\rho^r}
\]

which is the exact BP of the CDC-ROADM. This indicates that the influence of (A*) and (B*) on the analysis disappears when \( H = R \).

**C. Numerical Results and Discussions**

We first check the accuracy of the BP in (12) using simulation, where we employ the same parameters as those in Section III-C, except that the ROADM is a ROADM without FXC. As expected, Fig. 9 shows that the difference between the analytical results and the simulation results is quite small when \( \rho \) is not high and \( H < D \), and decreases with the increase of \( H \), and finally disappears when \( H = D \). This indicates our approximate result is accurate enough for the performance estimation of ROADM without FXC.

The results in Fig. 10 show that wavelength contention and Rx-resource shortage are the dominant factor of BP when the traffic load \( \rho \) is relatively low or relatively high. Also, for the ROADMs with the same \( H \), the load region where wavelength contention dominates when \( R = 160 \) is wider than that when \( R = 80 \). This indicates findings (f1) and (f2) hold in the ROADM without FXC.

![Fig. 9. BP of ROADMs without FXC vs. \( \rho \), where \( D = 8 \) and \( W = 40 \).](image-url)
We study the BP versus the number of drop ports $H$ in Fig. 11. We only consider the case where $H$ is the divisor of $R$. Assume $H$ can take $k$ integer values $H_1, H_2, \ldots, H_k$, where $H_1 = R/W$ and $H_k = R$. Let $P_{B,H}$ be the BP of a ROADM if the number of drop port is $H_i$, where $i = 1, 2, \ldots, k - 1$. We define $H_i$ as the saturation point of BP if $P_{B,H_i} / P_{B,H_{i+1}}$ is more than (95\%).

Fig. 11 shows that the BP declines and saturates with the increase of $H$. Similar to the one with FXC, increasing $H$ can reduce the BP of the ROADM without FXC when $R$ is large and $\rho$ is low, or when $R$ is large and $\rho$ is not high, since the BP is mainly induced by wavelength contention. On the contrary, the benefit of increasing $H$ is quite small when $\rho$ is low and $R$ is small, as Fig. 11 plots. That is, we observe the similar result as finding (f3) from Fig. 11.

Different from that of the one with FXC, the BP of the ROADM without FXC may saturate at an $H > D$, especially when $R$ is large and $\rho$ is not high. For instance, the BPs at $\rho = 0.75$, $\rho = 0.5$, and $\rho = 0.25$ saturate at $H = 10$, $H = 20$, and $H = 32$ when $R = 160$ in Fig. 11(b), respectively. This indicates that, to achieve the similar performance as a CDC-ROADM, a ROADM without FXC needs larger $H$ than that with FXC. In other words, the effect of installing FXC is similar to that of increasing $H$. We thus refine (f3) as follows.

(f3*) Installing FXC or increasing the number of A/D ports can reduce the BP incurred by wavelength contention.

In practice, we should consider the cost in system design. Large $H$ means a high cost for the ROADM without FXC. It is unnecessary to pursue the saturated $H$ in all the cases.

For example, in Fig. 11(b), when $H = 10$, the BPs at $\rho = 0.5$ and $\rho = 0.25$ are less than $10^{-8}$, which is already good enough for practical applications. On the other hand, FXC is also costly. Though the ROADM with FXC possesses contentionless feature when $H = D$, it requires an expensive $DW \times R$ FXC. We thus compare the ROADMs with and without FXC in terms of BP when $H = D$. As Fig. 12 shows, when $R = 80 = 25\% DW$, the BPs of two cases are almost equal, meaning that the FXC is unnecessary and $H = D$ is good enough. This is attributed to the fact that the shortage of Rx-resource is the main cause of request blocking. The situation is similar even when $R = 120 = 37.5\% DW$. When $R = 160 = 50\% DW$, the difference between these two cases becomes remarkable when $\rho$ is not high, and the FXC can remove the wavelength contention and thus reduce the BP in this case.

V. CONCLUSIONS

In this paper, we derive an exact BP and an approximate BP for the ROADMs with and without FXC, respectively. Our model can obtain the BP very fast, and our simulation verifies that the analytical results are accurate enough to predict the tradeoff between A/D flexibility and system complexity. We study the contention performance of the ROADM using the analytical results and have the following conclusions. For the CD-ROADM with a small number of RxS, the BP is determined by wavelength contention when the traffic load $\rho$ is low, and by Rx-resource exhaustion when $\rho$ is moderate and high. In this case, $H = D$ A/D ports are enough to achieve the best performance and it is unnecessary to install the FXC. For the one with rich Rx-resource, the BP is mainly caused by wavelength contention when $\rho$ is low and moderate. In this case, installing an FXC or $H > D$ drop ports is helpful to reducing wavelength contention and thus the BP as long as $\rho$ is not heavy.

REFERENCES


